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VERIFICATION OF QUANTITATIVE MAINTAINABILITY REQUIREMENTS

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FOREWORD

Various regulations, such as, AFR 80-5 and 66-29, and AFSCR 80-1 and 80-9 define policy for the reliability and maintainability (R/M) disciplines. Management specifications such as, MIL-R-27542 and MIL-M-26512, establish program requirements. In general, these documents do not present "how to" information which permits accomplishment of the essential program requirements.

The Technical Requirements and Standards Office (EST) is responsible for providing detailed guidance on R/M matters to all Electronic System Division (ESD) elements engaged in the acquisition of systems and equipment.

EST has published ESD-TDR-64-616, Handbook for Reliability and Maintainability Monitors, December 1964. This document, while containing specific techniques for demonstrating quantitative reliability, only briefly discussed the more involved problem of maintainability verification.

It is the purpose of this report to present quantitative methods associated with the maintainability discipline. As such, the report supplements the material contained in ESD-TDR-64-616.

Since maintainability techniques are expected to change, as results of current research programs become available, EST does not consider this document as a complete and final text.

George Allen
GEORGE H. ALLEN
ESTE

ABSTRACT

This document develops basic concepts for treating Maintainability quantitatively, with particular attention devoted to probabilistic aspects. It focuses on the special characteristics of the Lognormal Distribution as they relate to specifying and demonstrating numerical requirements. A catalog of Lognormal curves (both density and cumulative distribution functions) are included as well as recommended accept-reject criteria for Maintainability demonstration.

REVIEW AND APPROVAL

This Technical Report has been reviewed and is approved.

Frank E. Brandeberry

FRANK E. BRANDEBERRY
Colonel, USAF
Chief, Tech Rqmts & Stds Office

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ACKNOWLEDGEMENTS

If this report turns out to be a useful document, credit should go to several individuals - not simply the authors. First and foremost, credit should go to George Allen of ESD's Technical Requirements and Standards Office (EST) for recognizing that "Maintainability Demonstration" was a problem area that required study. His inspiring comments and direction caused this study to be conducted with an "open" mind without getting so abstract as to leave behind the original intent - namely to develop a technique that could be applied to Maintainability Demonstration of electronic equipment.

Special thanks are also extended to Sid Greenberg of The MITRE Corporation who suggested and arranged for the use of the Analog Computer Laboratory. Painstaking efforts by their personnel produced the graphs contained in the Appendix. These graphs must be considered an important part of this report since much of the analysis of the Lognormal distribution was accomplished by visual inspection of changes to the curves as the parameters were varied. Certain analytical results rest heavily on the accuracy of these graphs also.

One final note of gratitude is made to Jerome Horowitz, also of EST, and Jerome Klion, Rome Air Development Center (RADC), Rome, New York, both of whom provided a patient ear and helpful comments whenever things needed to be "talked out" prior to going to the "drawing board".

CHAPTER I

INTRODUCTION

In our view, the quantification and verification of Maintainability has been at a standstill since 1962 when Appendix A of MIL-M-26512B was published. This, we feel, was caused by the fact that methods given in Appendix A went too far and too fast before basic concepts were established. Most practitioners, although they failed to grasp the recommended methods, were reluctant to show their ignorance and merely tried to "make a fit" using the "poorly constructed shoe." This problem was compounded by the fact that the highly complex lognormal distribution was chosen, and few people had more than a scanty knowledge of how to apply this distribution. One of our aims shall be to correct this unfortunate situation.

In making the study contained in this report, our task seemed, at times, to be an impossible one. We started making progress when we realized that no less than <u>three</u> problems were being dealt with--quite often, simultaneously--and it was not always easy to separate one from the others. One of these problems has been mentioned already, namely, that of explaining basic concepts, with particular emphasis on what must be known about the basic distribution (the lognormal, say) before one attempts to use it as a mathematical model.

The other two problems are:

(1) Most people are nurturing <u>erroneous</u> concepts which resulted from using methods that were not understood. These, of course, were held by those who had originally developed these methods. Hence, our explanations could not always follow a smooth road--we would often have to take detours to destroy false notions.

(2) Most people confuse "estimation" with "demonstration." (For this reason, we have used the word "verification" in the title of this report.) This problem may be stated another way: Previous attempts to solve this problem gave "estimation" methods to solve a "demonstration" problem. Naturally, such fallacious thinking would have to be corrected before <u>our</u> recommended <u>demonstration</u> method could be understood.

Since this is an <u>educational</u> problem as much as it is a <u>technical</u> problem, much of this report is written in tutorial style. If, because of his background, the reader finds certain passages too elementary in nature, we beg his forgiveness and ask him to skip to those portions which hold his interest. Those same passages which he finds dull, will probably seem rather difficult to those readers who are still novices in this subject.

To save the "expert" time, we shall describe the configuration of the remainder of this report. Chapter II, "Survey of the Problem," does just that. It first forces us to focus on the aspect of Maintainability that causes most of the difficulty, namely, the <u>probabilistic</u> aspect. After a brief discussion of the connection between Maintainability and Probability (via the notions of "time-to-repair" and "average-time-to-repair"), Chapter II then points out what factors must be considered in order that this connection brings Maintainability and Probability into a close and intimate union. Chapter II concludes by bringing out those features of the lognormal distribution which make it a "natural" choice for the distribution function; but then Chapter III goes on to show why previous methods for specifying Maintainability requirements (in light of this distribution) were in error, and shows why this distribution does not adequately serve us as a design criterion. Specifically, Chapters II and III build up to the following main points:

(1) Both the Exponential and Normal distributions must be rejected entirely as math models for Maintainability.

(2) Although the lognormal distribution may appropriately describe systems and equipment, as they are usually built, it is too restrictive (has too many undesirable characteristics) to serve us as a design criterion for future use--the way we want them to be built. This is especially true for the new "automatic switch-over" type systems.

(3) Lacking evidence to the contrary, we may be obliged to specify requirements in light of the lognormal distribution, in which case the "Mean" and the "95th percentile[1]" are not enough to fix the nature of this distribution and should not be used in specifying requirements in contracts, as was previously recommended. It is specifically recommended that the Median and the 90th percentile be used to specify requirements. It is recognized, however, that Availability requirements are often used to dictate Maintainability requirements and, under these conditions, only the Mean is known. Although this does not change the fact that the Mean does not give the contractor sufficient guidance, we have presented, in Chapter III, a table for translating a requirement for the Mean into a requirement for the Median and the 90th percentile.

Chapter III contains the main results of this study; hence, this chapter serves as an introduction to Chapter IV, which then treats the problem of demonstration in view of (and in spite of) the special characteristics of the lognormal distribution. Specific decision (accept-reject) criteria are presented as well as methods for arriving at quantitative Maintainability requirements which are compatible with the given accept-reject criteria. Appendix A contains many graphs of the lognormal distribution (both the density and cumulative distribution function) which, we believe, are not available anywhere else.

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[1]Other authors have called the 95th percentile "Mmax." In this report we deal with the 90th percentile which we call Xmax, but everything we say about Xmax shall hold, in substance, for Mmax as well.

It should be noted that "time-to-repair" is treated as an undefined term throughout this report. This means that we hope our findings would appertain regardless of the kinds of "down-time" that would be included in the "time-to-repair" statistic. It also means that we hold to the belief that "time-to-repair" must be defined by the procuring activity for each procurement in cognizance of such factors as:

(a) Those aspects of Maintainability which are controllable by the contractor and within the purview of the contract. For example, for certain procurements (not strictly "off-the-shelf") the amount of development that is consistent with the cost-framework provided for that procurement is limited. Thus, it may not be reasonable to include "all" types of downtime; i.e., only diagnostic time, time to isolate, actual repair or replacement time, and check-out time could be included in our definition of "time-to-repair". In other cases, we may feel that it is reasonable to include other kinds of "down-time" such as travel time, as part of our definition of "time-to-repair".

(b) Where and when the demonstration is to take place. If it is important to gain assurance that Maintainability requirements are satisfied before the equipment is shipped and installed at the operational site, then in-plant demonstration must take place. Any aspects of "down-time" which cannot be duplicated in an in-plant environment would have to be excluded from our "time-to-repair" definition.

Some have tried to resolve the "definition dilemma" by placing adjectives in front of the word "downtime", such as "corrective maintenance downtime" or "active corrective maintenance downtime", etc. This does not solve the problem at all--it merely serves to make the procuring activity not mindful of his responsibility to precisely define these words. It is our opinion that such adjectives should be dispensed with altogether--the added jargon may be useful for conversational purposes but it is simply added confusion in a contractual situation.

Another advantage in leaving "time-to-repair" undefined is that it may turn out that what we say about this statistic may also hold for "time-between-maintenance actions" (MTBM) for those systems/equipment where preventive maintenance is an important factor. Although this report does not explore this possibility, it is easy to see that our demonstration method could apply to MTBM as well.

Before leaving this introduction we would like to give some final thoughts that lay in the back of our minds, as we made this study. We believe that previous attempts at the quantification and verification of Maintainability tried to follow the same road that was travelled so successfully in the discipline of Reliability. For Reliability, this road had the following superficial appearance[1]:

---

[1]Statistical terminology used in this introduction is fully explained in subsequent chapters.

(a) Choose a random variable (time-between-failures).

(b) Select a probability distribution which governs the behavior of that random variable (Exponential Distribution).

(c) Specify quantitative requirements in terms of the key parameters of that distribution (the mean or mean-time-between-failures).

(d) Use standard statistical procedures to verify those stated requirements (that is, the key parameters of the distribution).

Since the "time-to-repair" statistic fill the bill as a random variable, the above road-map seems to tell us that all we need is a probability distribution to reach our destination. Two such distributions have been suggested, and even used, namely, the Exponential Distribution and the Log-normal Distribution (occasionally, we have seen references to the Normal Distribution also).

We feel that this report shows that the problem is somewhat more complex than stated above. We hope to prove that even if the "distribution problem" was solved (and some are attempting to gather empirical evidence for this purpose) there are conceptual difficulties which need to be worked out--and not camouflaged by mathematical "juggling". Moreover, we feel that these problems are not going to be solved by statisticians alone--but by engineers since they arise when we try to marry our mathematical model to the real-world Maintainability problem. Roughly stated, the critical problem is this: How do we get contractors to deliver us a "better" product, from a Maintainability standpoint, within the cost framework provided for the procurement?

People tend to forget that these problems were once present in the development of Reliability (in fact, some[1] say that they are still there). Efforts to explore these conceptual difficulties or to develop an appropriate mathematical model for Maintainability have been, to our knowledge, practically non-existent. We hope that this report will help to remedy this situation by consolidating certain widely held notions, while destroying other notions which are either erroneous or are retarding progress. However, if this report merely serves to inspire others to express their ideas and proposed solutions, we shall feel quite satisfied.

[1]For example, Proschan, Barlow, and others are making a strong bid to replace the concept of a constant failure rate by a monotone failure rate.

CHAPTER II
SURVEY OF THE PROBLEM

1. Preliminaries:

Before we start talking about "Maintainability" it might be wise to explain what we mean when we use the word. We shall not attempt to give a universal definition - one that is advocated whenever this term is used. If we gave such a definition, its generality would render it useless for demonstration purposes. For example, AFR 66-29, dated 27 April 1964, defines Maintainability as

> "A characteristic of design and installation which is expressed as the probability that an item will conform to specified conditions within a given period of time when maintenance is performed in accordance with prescribed procedures and resources".

This statement has the appearance of being deliberately broad - in fact, at first glance it seems to be applicable to any design characteristic. If we tried to use this definition for specifying (and subsequently demonstrating) Maintainability requirements, we would probably be frustrated by the number of words or phrases that are left undefined. For example, much groundwork is needed before the word "probability" has an unambiguous meaning. Moreover, in particular circumstances certain phrases may not have any significance at all; to wit, "period of time" for a continuously operated radar system.

These remarks should not cause the reader to lose faith in Air Force regulations. At best, regulations merely give the general usage of certain technical terms, not precise definitions. The so-called "definition" is there for our consideration and deliberation; it is not to be taken literally. It is not even expected that one must provide a counterpart for each phrase appearing in the definition.

What is done instead? We shall not keep the reader in suspense any longer. For particular procurements, one must first consider Maintainability in a broad sense (considering for example, its impact upon Availability) in order to find crucial elements. These elements are derived through carefully considered intuitive judgment, as well as experience and knowledge gained on systems or equipment of like kind, being particularly careful to choose elements that may be precisely defined. These crucial elements are often peculiar to that equipment only and may not cover the entire Maintainability picture. For example, suppose the following statement appeared in a work statement:

> "The operational equipment shall be designed for ease of maintenance".

Although this statement covers a big chunk of Maintainability, and seems to be an appropriate statement for all procurements, it doesn't say what exactly is required; hence we may have difficulty demonstrating that this requirement has (or has not) been met. Now, consider the following approach:

"The operational equipment shall be designed for ease of maintenance, containing as a minimum the following features:

(i) Components or units listed in Section ____, whose expected failure rate exceeds _____ hours must be accesible for repair without requiring the removal of other components (which must be replaced after correcting the malfunction) unless such removal can be made in less than ____ minutes.

(ii) Each functional module (as defined in Section ____) must contain at least one test point which will provide for the measurement of the critical parameters of that module.

Here, we are assuming that each of these items is practical and important for the system being considered. For that system, these specific items may not cover the full spectrum of Maintainability as expressed by the phrase "ease of maintenance;" however, we have focused upon certain crucial elements and we have cited <u>specific</u> requirements that are <u>demonstrable</u> either during design reviews, in-plant testing, or field testing.

We need not add anything further about the demonstration of "deterministic" requirements such as (i) or (ii) above. Compliance (or non-compliance) with such requirements is obvious; e.g., either a test point exists for each functional module or it doesn't.

Most of this report shall be concerned with "observables" that are somewhat more complex - that is, observables that are more closely allied to the definition given in AFR 66-29. More specifically, in subsequent paragraphs, we shall try to develop certain techniques for dealing with the concept of "average-time-to-repair", which is directly related to the notion of "probability".

For more information concerning the deterministic aspects one may consult MIL-STD 803 and ESDP 80-9.

2. Probabilistic Aspects of Maintainability:

We find that deterministic aspects are not enough to insure that equipment has the required degree of Maintainability. The following question cannot be answered deterministically: "How long on-the-average does it take to fix the equipment once it fails?" In fact, before attempting to answer this question, one must decide what is meant by "average".

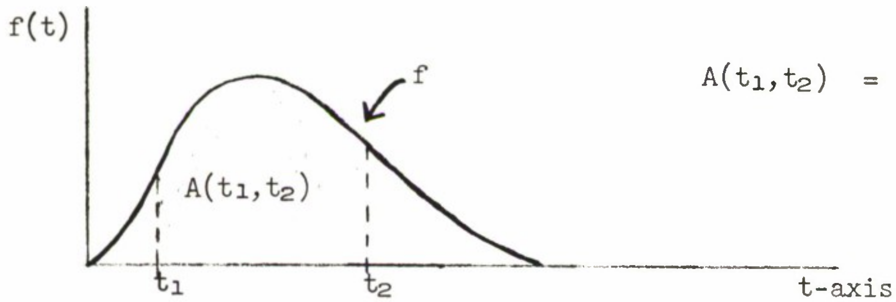
Anyone who has had dealings with "averages", or has taken just one course in elementary statistics, should know that such descriptions can be misleading. There are several kinds of averages: the arithmetic mean, the geometric mean, the median, the mode, to mention just a few. A fundamental problem is to choose the right one.

However, we cannot make an intelligent selection without <u>first</u> considering the question:

"How are the various values of repair-times distributed probabilistically?"

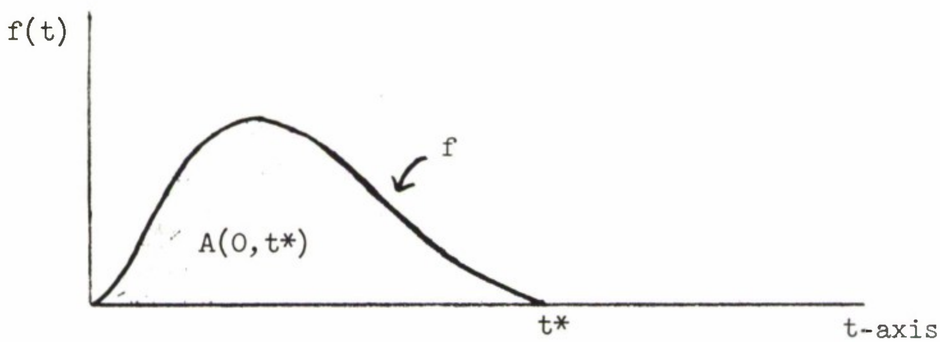
This is quite a mouthful! - so let us try to explain what this means. To do so requires some knowledge of probability theory; hence, we must make a slight detour to be sure that the reader is "in-tune" with us.

We start by considering a curve f, shown below, and a function A. The function A gives the area under the curve f between certain limits on the t-axis (time axis).



$A(t_1,t_2)$ = Area of shaded portion

Now, suppose the total area between the curve and the t-axis is 1. Then (see picture below) the area between 0 and t^* is equal to 1, and if $t_1 \geqq 0$ and if $t_2 < t^*$, then the area between t_1 and t_2 must be less than 1; that is $A(t_1,t_2) < 1$ in the picture shown above.



Now, suppose we also knew that $A(t_1,t_2)$ could be converted to a probability statement as follows:

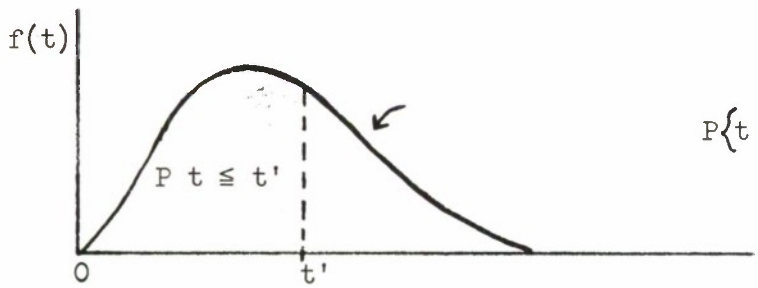
$$A(t_1,t_2) = \left\{ \begin{array}{l} \text{The probability that a repair} \\ \text{time } t \text{ will take between} \\ t_1 \text{ to } t_2 \text{ hours to complete} \end{array} \right\}$$

Symbolically, we could write the above probability statement as

$$P\{t_1 \leq t \leq t_2\} \quad \text{so that}$$

$$A(t_1, t_2) = P\{t_1 \leq t \leq t_2\}$$

We would then know, as depicted below, that $A(0,t') = P\{0 \leq t \leq t'\}$



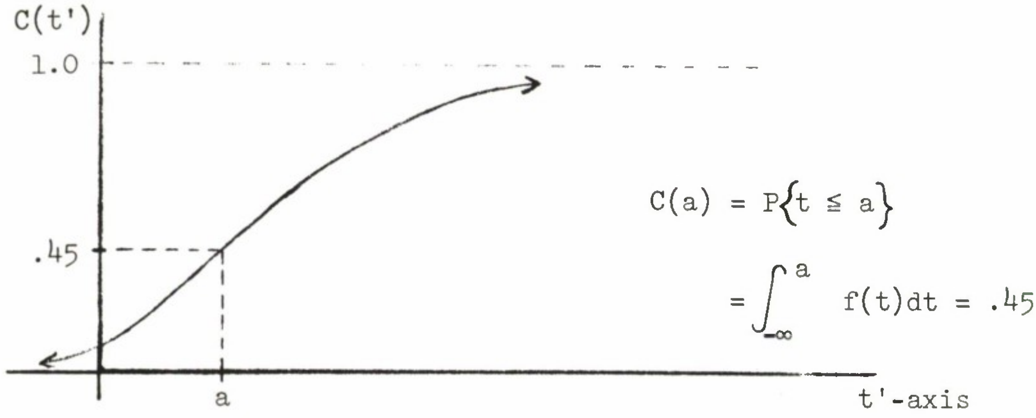
$P\{t \leq t'\}$ = Area of shaded portion

and since we know that the lower limit of a repair time is at least 0, we may state $P\{0 \leq t \leq t'\}$ more simply as $P\{t \leq t'\}$. In words, the area under f from 0 to t' is equivalent to the probability that an arbitrary repair time will take less time than t', which establishes what is commonly called a "cumulative distribution function", which we shall call C. More generally, the function C, for <u>any</u> function f, gives the cumulative area under the curve f from $(-\infty)$ to time t'. Thus;

$$
\begin{aligned}
C(t') &= A(-\infty, t') \\
&= \int_{-\infty}^{t'} f(t)dt \\
&= P\{t \leq t'\}
\end{aligned}
$$

The curve f, which C is dependent upon, is usually called a "probability density" function. The variable t has a special name also --- "random variable".

The cumulative distribution function always has the following form (which may be shifted right or left, or climb more rapidly, depending on the function f):



and exists for any density function f. Note that this function is non-decreasing, that is, $P\{t \leq t'\}$ gets larger (closer to 1) as t' gets larger (since, with larger t', we are taking more area under the curve f). The reader should also observe that numerical values have been indicated on the vertical scale showing that C(t') is always between 0 and 1. For the density function f, the values on the vertical scale have no probabilistic or physical meaning, since we are merely interested in the area under the curve; that is, the vertical scale simply gives values of f(t) which make the area under the curve equal to 1.

3. Some Required Properties of the Density Function:

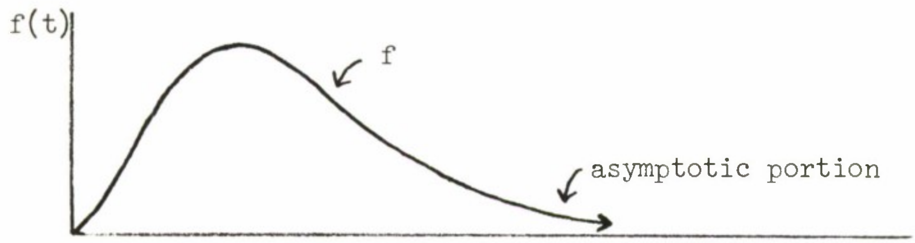
Once we know the function f or C we then say that we know how our random variable (repair-times) is distributed probabilistically. Whenever statistical data exhibits "chance fluctuations", (i.e., any particular result is unpredictable) it is essential that we consider the distribution that gives rise to these results in order to give meaning to our concept of "average value". Our present aim shall be to determine the general shape of the density function for electronic equipment (or, rather, what it should be), recognizing, of course, that there will always be exceptions to any rule that is established. Hence, we must now devote our discussion to the selection of a "generalized" density function[1], with particular emphasis on the reasons therefore. We shall develop the notion of "average value" as the discussion proceeds. Hopefully, this will remove some ambiguities when the expression "average-time-to-repair" is used.

The function f that was illustrated previously does not meet our needs for several reasons. First, we do not have the mathematical expression for this curve which causes trouble when we make analytical investigations. Second, for nearly any interval on the t-axis (however far to the right it may be) there is a chance (however small) that a repair time could fall within that interval. Occasionally, equipment might require several weeks before the cause of a malfunction is found[2]. Our previous curve f, however,

1 The comments of this paragraph shall be made precise after we introduce some terminology; namely, that of a "class of functions".

2 We are assuming, of course, that our "repair-time" definition includes diagnostic time.

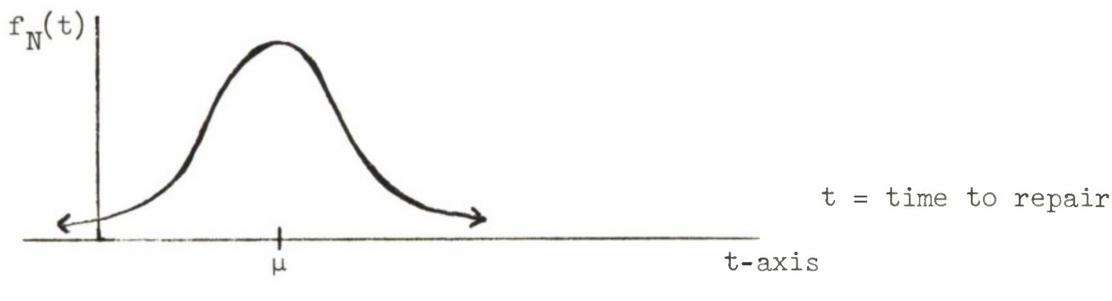
does not go beyond t*. However large t* is, it might not be large enough. Happily, this defect is easily corrected by using a certain mathematical concept; namely, the concept of a curve "asymptotically approaching" the t-axis (see picture below):



Thus, no matter how far to the right we move on the t-axis, our curve never touches the t-axis; but for any interval of fixed size, the further we move the interval to the right (in the decreasing portions of the curve) the smaller the area under the curve will be. In Calculus we learn that there are many curves of this type, which can be expressed mathematically, having a finite amount of total area beneath them. With only minor modifications (i.e., multiplying such functions by an appropriate constant in order to make the area under the curve equal to 1), any of these functions would qualify, mathematically speaking, as our density function. One of our major problems is to find a class of functions, which best describes equipment encountered in ESD procurements. We shall now discuss some examples in order to clarify what we mean by "class". Consider the function defined by the rule

$$f_N(t) = \frac{1}{\sigma\sqrt{2\pi}} \quad e^{-\frac{1}{2\sigma^2}(t-\mu)^2}$$

where t represents "time-to-repair", and e = 2.71828...etc. This is the well known "normal curve", having a bell-shape appearance



i.e., it is symmetrical about its highest point. The two symbols μ and σ that appear in the mathematical expression are constants which, when known, "fix" the "location" and "specific shape" of the curve as follows. The constant or parameter μ tells us exactly where (on the horizontal scale) the point of symmetry occurs. The other parameter σ tells us how "flat" the curve is; i.e., is the curve highly concentrated about μ or is it rather "spread out"? (The technical terms for μ and σ are the "mean" and "standard deviation" which shall be given precise definitions later).

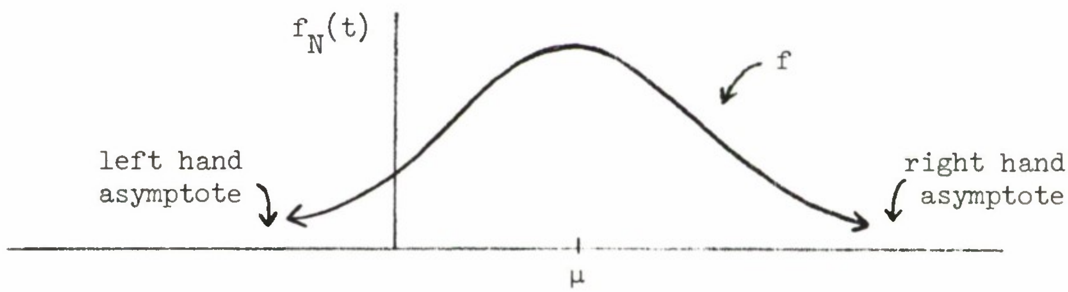
Clearly, μ is most representative of our concept of average-time-to-repair as far as the normal curve is concerned. In fact, we shall find that our concept of "average" is always represented by a "location parameter". We should also observe that changes in μ or σ give distinctly different curves, and bring significant changes to our cumulative distribution function which in this case is denoted by

$$C_N(t') = \int_{-\infty}^{t'} f_N(t)dt$$

Thus, $f_N(t)$ describes a collection of curves, each having a bell-shape appearance. We shall refer to such a collection as a "class of functions"; for this case it would be called the normal class.

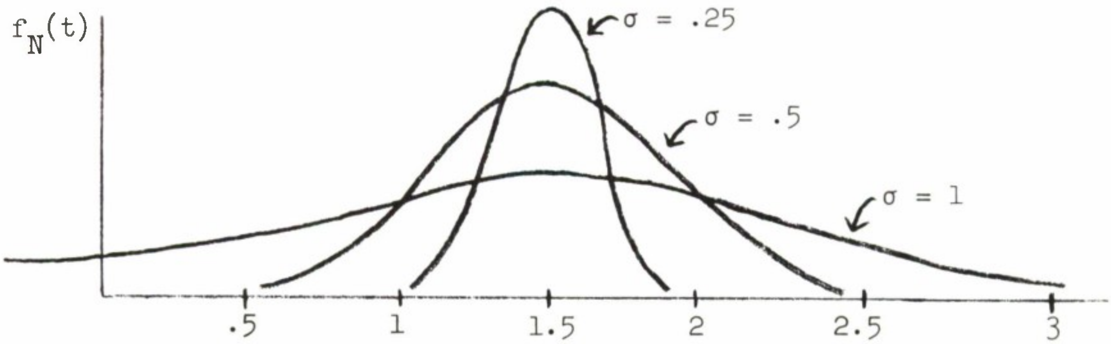
4. Undesirable Properties of the Normal Class:

Can the normal class be used to represent our density function for time-to-repair? Does it appropriately describe the probabilistic behavior of our repair-time statistic? It would be nice if it did, since we would have no trouble in giving meaning to our concept of "average" repair-time...it couldn't be anything else but μ. However, other problems arise, which might cause extreme amounts of difficulty. For one thing, although the normal curve has a right-hand asymptote, which is a desirable property, we find that it also has a left hand asymptote which is quite undesirable! This is depicted below.



Realizing that negatively valued repair-times have no physical meaning, we would not like to see any area under the curve to the left of the vertical axis. The reason this feature is distasteful to us is that for a given σ, the lower the value of μ the more area will have to be "thrown away". What this means in a real situation cannot be mentioned at this point in our discussion - we will return to this point later when we discuss the development of a demonstration model against which test results must be compared.

The normal class has other problems which will become obvious in the following example. Let us suppose that 1.5 hours is a minimum "average-repair-time" requirement for a particular radar set. In addition, we would like to specify requirements for certain percentiles, e.g., that 25 percent of total repair actions will take less than 1 hour and 98 percent will take less than 2.5 hours. Is there a normal curve that has these features? We shall show that these requirements are talking about a non-normal shape, which will bring out the fact that our preference for one class of functions, rather than another, is based mostly on two features: (1) the asymptotic properties and, more important, (2) the shape that results from varying the parameters. Let us now examine some functions in the normal class, all with 1.5 hours as their mean, with σ = .25, .5, and 1, as depicted below:



NOTE: For those mathematically oriented, we mention here that the points of inflection, of the normal class, $(d^2y/_{dt^2} = 0)$ are always at $\pm\ \sigma$ distance from the mean (1.5), and that 68% of the area falls between these limits.

We now point out that for the normal class, 95% of the area must fall between $\pm\ 2\sigma$ units distance from mean, with $\pm\ 3\sigma$ corresponding to 99% area. Thus, for a mean of 1.5 hours, if the normal class is applicable, we see that a requirement that (say) 97 1/2% of all repair times be completed in less than 2.5 hours would mean that only 2 1/2% could take less than .5 hours, and only 13.5% would be completed in less than 1 hour!

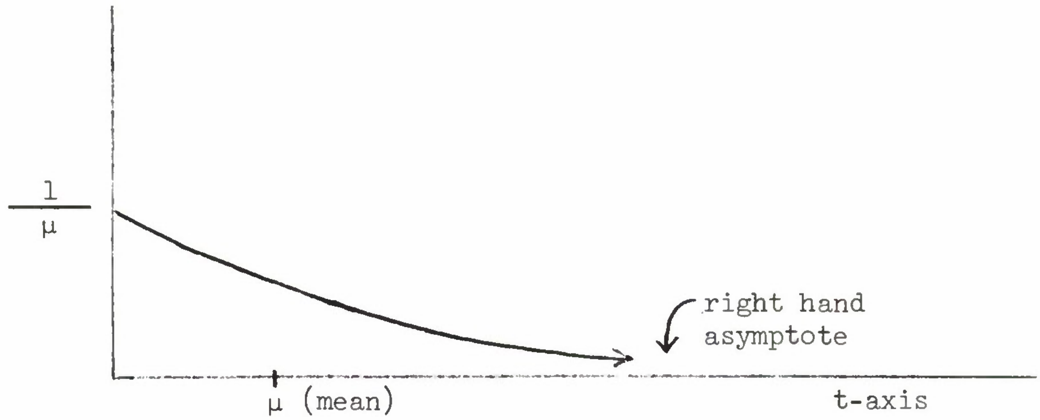
This is quite unrealistic. Our intuition would cause us to believe that equipment could be designed to "look good" at both ends of the scale. Hence we seek a class of functions that has such an appearance, as the shaping and location parameters are varied. If such a class exists, it will better serve us as a design criterion, and subsequently as a demonstration model.

5. Undesirable Properties of the Exponential Class:

One possible candidate that has been suggested from time to time is the so-called "exponential class" with density function:

$$f_E(t) = \frac{1}{\mu} e^{-t/\mu}$$

with shape:



We note that this function "looks good" at both ends; however, we shall find that it is "too good to be true". In order to speak precisely about this and subsequent classes of functions, we require certain definitions of terms which completely determine any density function f(z). These are:

The Mean: $M\{z\} = \int_{-\infty}^{\infty} x\, f(x)\, dx$

The Variance: $V\{z\} = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\, dx,$ $\left(\text{where } \mu = M\{z\}\right)$

The Standard Deviation: $S\{z\} = \sqrt{V\{z\}}$

The Median: $M^*\{z\}$ is that value t at which

$$\int_{-\infty}^{t} f(x)\, dx = 50\%$$

The Mode: $M_o\{z\}$ is the value at which f(z) attains a maximum.

When it is perfectly clear what random variable we are talking about, we shall not use the cumbersome notation, $M\{z\}$, prescribed above. Instead, we shall abbreviate the notation as follows:

$M\{z\} = \mu$

$V\{z\} = \sigma^2$

$S\{z\} = \sigma$

$M^*\{z\} = \theta$

$M_o\{z\} = \xi$

It should be observed that for the normal class, the mean, median, and mode were all equal to each other.

Returning now to the exponential class,

$$f_E(t) = \frac{1}{\mu} e^{-t/\mu}$$

one easily verifies that the mean is μ, that the variance, σ^2, is equal to μ (hence the standard deviation, σ, is equal to $\sqrt{\mu}$), and the median, θ, is equal to $(-\mu)(\ln .5)$, where "ln" denotes the natural logarithm function. The mode, of course, is zero, a feature which leads to certain difficulties, as will be seen shortly.

Since σ and θ are both functions of μ alone, knowledge of any one of these parameters is equivalent to knowledge of the other two; therefore, one usually thinks of the exponential class as a "one-parameter" distribution. Whether this characteristic is a realistic stipulation for the "time-to-repair" statistic is highly questionable, despite the simplicity that results from having only one parameter to worry about.

On the other hand, for this density function we have at least two "location parameters", μ and θ, which have meaning. Which one is most representative of the "average" value? Perhaps it matters little which one is chosen, since fixing one fixes the other. In the discipline of Reliability, where the exponential class is widely used, the mean has won out over the median. In fact, one reason for the occasional use of the exponential function for Maintainability was that it seemed to be a "natural" choice in view of the following correspondences:

	Reliability	Maintainability
Event of Interest:	Failure	Repair
Random Variable:	Time-Between-Failures	Time-To-Repair
Distribution:	Exponential Class	Exponential Class
Key Parameter:	Mean-Time-Between-Failure, μ, or Failure Rate, $1/\mu$.	Mean-Time-To-Repair, μ, or Repair Rate, $1/\mu$.
Mathematical Definition:	$R(t) = e^{-t/\mu}$ $= \left\{\text{Probability of failure-free operation for time } t\right\}$	$M(t) = 1 - e^{-t/\mu}$ $= \left\{\text{Probability of completing a repair by time } t\right\}$

What could be simpler? We have now, supposedly, quantified maintainability, and may now use the statistical procedures (already developed in the field of Reliability) for its measurement. What could go wrong?

Quite a few things could go wrong. The fundamental problem is this: One must do much more than create an idealized model of a physical situation. The model must pertain to the problem at hand, and there should be some empirical evidence to justify the chosen class of functions. In addition to empirical evidence, there should be intuitive reasons for the choice; or, stated another way, there should not exist good intuitive arguments <u>against</u> the chosen class. There are three such arguments against choosing the exponential class for Maintainability:

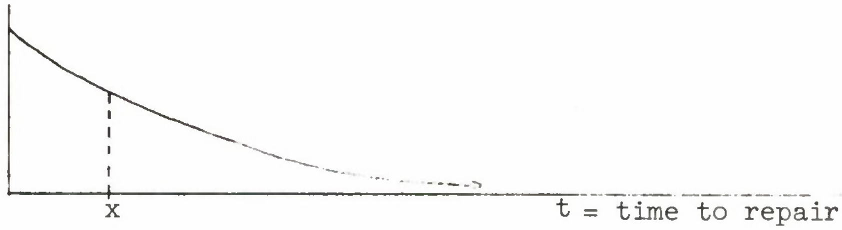
(1) For the exponential class, the probability of completing a repair within the next t hours is the same no matter how much time has elapsed since the repair action was started. Very few people would accept the truth of this statement. In fact, some people question its validity for Reliability as well, for which the statement reads: The probability of no failures in the next t hours is the same no matter how much failure-free operating time has been accumulated. This feature of the exponential class, which is <u>not</u> present in the other classes that we shall consider, may be stated in mathematical terms as follows: Define the "repair rate function", h, by the relation

$$h(t) = \frac{f(t)}{1 - C(t)}$$

where f is the density function and C is the cumulative distribution function. It will be found that h(t) is a constant function for the exponential class, but is not constant for the other classes of functions that we will consider.

(2) For the exponential case, equipments with the same mean-time-to-repair, have exactly the same distribution of repair times. In other words, the mean-time-to-repair completely describes the distribution and it would not be possible for equipments with the same "mean" to have a significant difference in their variances. This feature, even more so than (1) above, runs head-on with the intuitive judgment of most people.

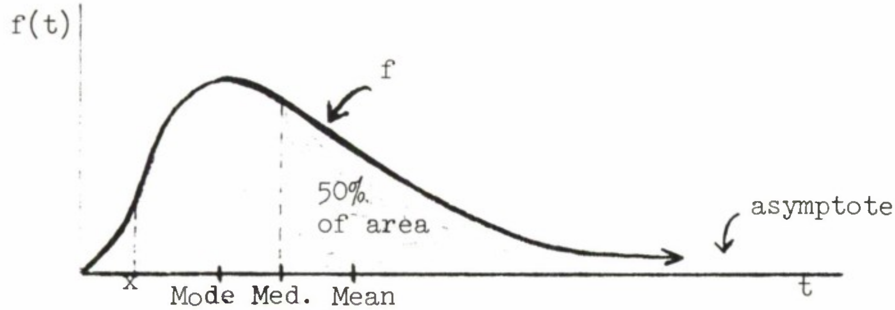
(3) Since the exponential density function takes the following form:



we have a considerable amount of the total area to the left of the point x shown above (the shaded area). Since this area gives the probability that a repair will be completed in less time than x, we see that the exponential function would often give undue weight to short repair times; i.e., one must move x rather close to the origin before the probability of completing a repair in less time than x becomes small.

6. <u>Desirable Properties of the Lognormal Class</u>:

Some might say that such problems are trivial - that they are immediately solved by changing the function to one that coincides more closely with people's intuitive tendencies. As a matter of fact, such a function has been found. It is backed by both empirical evidence and intuitive judgment, and the three problems mentioned above are no longer present. Chapter III discusses this function in detail, and Appendix A contains several graphs for various parameter values. Typically, the curve has the following appearance:



that is, it starts at $f(t) = 0$, and once it starts its climb, it does so rather rapidly until reaching its highest point; but after attaining its maximum, the descending portion is never as steep as the ascending portion. Thus there is no left hand asymptote, but a right hand asymptote exists, as desired, and (depending on the standard deviation, of course) one does not have to move x very "close" to zero (see picture above) before the area from zero to x is small (and consequently the probability of completing a repair is less time than x is small).

The above picture also shows that there are no less than 3 location parameters, the mean, median, and mode, one of which is to be chosen to represent the "average value" (average-time-to-repair) and they always have the following relationship:

$$M(t) > M^*(t) > M_o(t)$$

that is, the <u>mean</u> is greater than the <u>median</u> which, in turn, is greater than the <u>mode</u>, <u>always</u>, no matter what standard deviation may be. We might find that <u>neither</u> of these location parameters is best representative of the "average value" for <u>all</u> occasions; that is, the "best one" may vary with (say) the intended use of the equipment or other factors.

7. Summary:

Now that our survey has led us to the Lognormal class which, on the surface at least, exhibits no undesirable properties, it may seem that our task is completed. Actually, our study begins at this point because of the "not-so-obvious" shortcomings of the Lognormal class which shall be covered in detail in the next chapter.

Hence, this chapter has merely surveyed the problem up to the point where our efforts began to bear fruit. It is therefore rather easy to summarize the present chapter - it told us that our problem reduced to making a determination of (1) how to specify requirements in light of the Lognormal distribution and (2) how to verify that such requirements have been met. In the process of trying to answer these questions we derived some surprising, if not important, results all of which are contained in the next chapter.

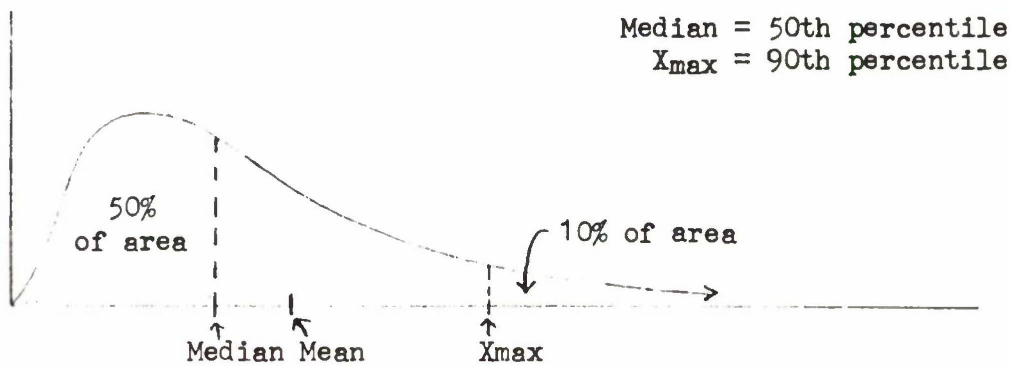
CHAPTER III

THE LOGNORMAL DISTRIBUTION - SOME IMPORTANT RESULTS

1. Practical Considerations:

Although we have removed most of the difficulties associated with the normal and exponential class, we have introduced new problems which partly arise from having to deal with a two-parameter distribution. Before this distribution is completely specified, we must know at least two of the following: the Standard Deviation, the Mode, the Median, or the Mean. There is also an educational problem, since elementary statistical courses and textbooks do not usually treat this distribution and its special properties are, generally speaking, not known by those responsible for Maintainability programs. Therefore, the remainder of this chapter shall attempt to describe these special properties--in the process we reinterpret these properties from a Maintainability viewpoint, particularly with regard to their effect on specifying and subsequently demonstrating quantitative requirements.

Because practitioners in the field have become accustomed to specifying the Mean and an upper percentile (e.g., the 95th percentile), we shall explain the characteristics of the Lognormal class in terms of such quantities. Let us first describe what we are talking about via the following illustration:



The Median is called the 50th percentile because 50% of the area under the curve lies to the left of that point. Similarly, the expression "X_{max}" is used to denote the 90th percentile. We may say, equivalently, that the probability of completing a repair in less time than the Median value is 50%, and the probability of completing a repair in less time than the X_{max} value is 90%. Some may prefer to think of X_{max} in terms of "relative frequency"; that is, over the long run, only 10% of repair actions will take more time than the X_{max} value.

It should be understood that the Mean value has no simple relationship with the notion of area (the cumulative distribution function); that is to say, one must know much more than the Mean value to derive probability statements of the kind just mentioned. It might be suspected, then, that for the Lognormal distribution, the Median and Mean should not simply be thought of as "measures of location"--they seem to be telling us quite different things about the nature of the curve. While the Mean (alone) may tell us more about the mathematical properties of the curve (i.e., the density function), the Median (alone) tells us more about the probabilistic nature of the curve (i.e., the cumulative distribution function). Since there are severe restrictions on the probabilistic nature of the curve to the left of the Median (as pointed out in Chapter II) knowledge of the Median value is almost equivalent to knowledge of this entire portion of the curve. Thus, it appears that if Xmax was known, which tells us quite a bit about the right hand portion of the curve, practical people would prefer to know the Median value, rather than the Mean value, if only one of these could be obtained. We shall provide more justification for this statement in the sequel.

We will show, in fact, that we give prospective contractors much better guidance when we specify the Median and Xmax values, instead of the Mean and Xmax values.

The Mean is best described by thinking of the plane area under our curve as a visual representation of a unit mass (of uniform density) which is bounded by the curve and the horizontal axis. With this conceptual framework in mind, the Mean is that point, on the horizontal axis, at which this mass would be in perfect balance. In this way we see that the Mean is rather heavily affected by the asymptotic portion of our curve; that is, long repair times, even though they may rarely occur, heavily affect the Mean value.

Because there is no simple relationship between the Mean value and the cumulative distribution function, the curves in the Appendix were developed by varying the Median values and the Standard Deviation. However, the graph on the next page (Figure 1) gives us all the information we shall need to analyze how changes in the Mean value affect given curves also.

We are now ready to explain the most important findings of this study which are mostly derived from analyzing the Median curves in Figure 1. For this reason, we suggest that the reader become intimately familiar with Figure 1--what it represents and how it was derived. The next two paragraphs explain how Figure 1 was constructed.

Both axes are expressed in minutes. The horizontal axis represents Mean values, while the vertical axis represents values of Xmax (the 90th percentile). Every pair of values that are plotted (shown by a little circle) was taken from a particular density curve in the Appendix and therefore represents a different density function. (The data points are tabulated in Table 1.) We have taken the liberty of connecting those data points that have the same Median value by a continuous curve, and then

Figure 1

MEDIAN CURVES DERIVED FROM PAIRED
MEAN AND Xmax VALUES

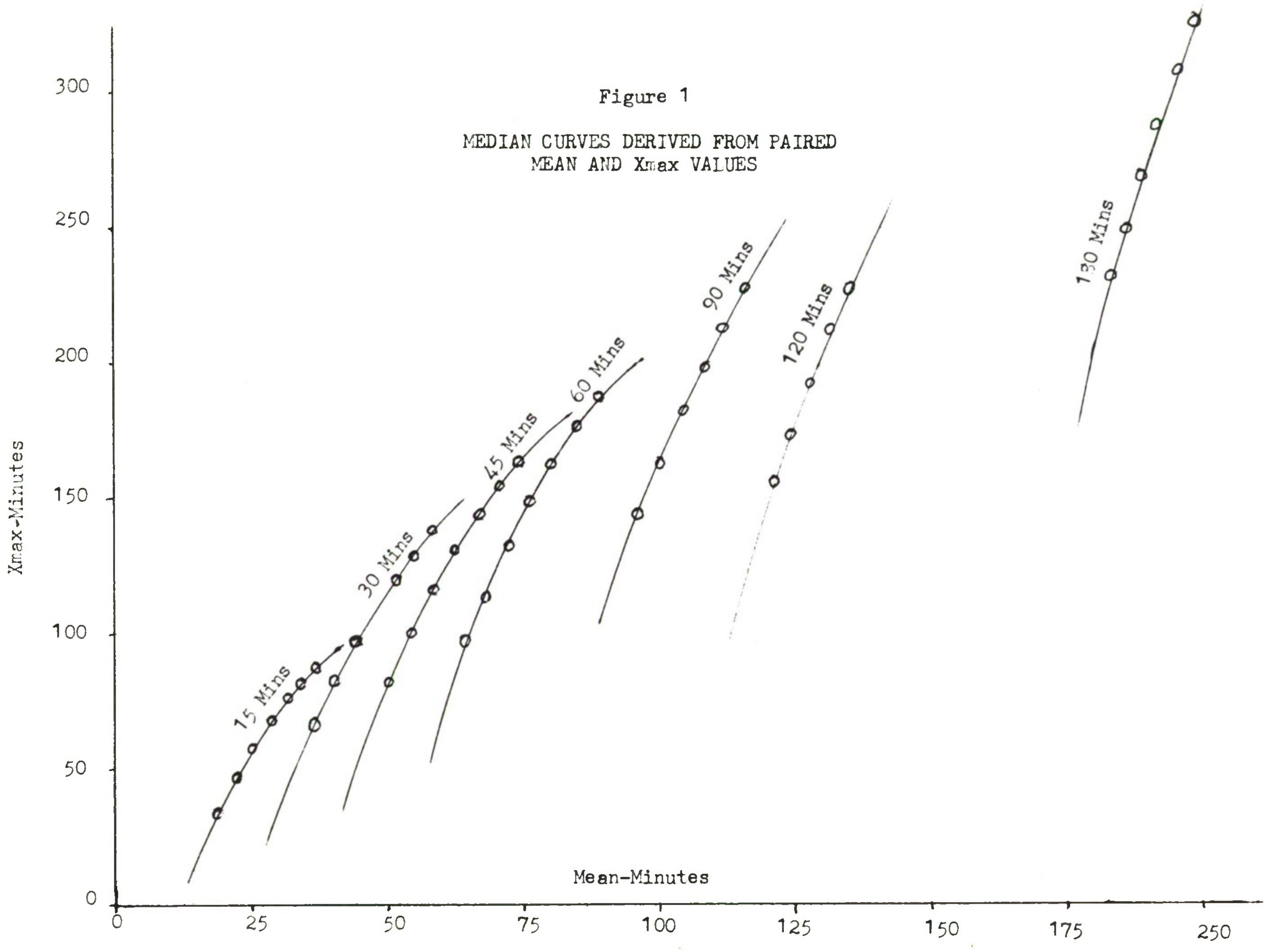


TABLE 1

DATA POINTS FOR FIGURE 1

Median: 15

Xmax	Mean
33	18
47	22
59	26
68	29
76	32
82	35
88	37
94	40

Median: 30

Xmax	Mean
67	36
83	40
97	44
119	52
129	55
137	58

Median: 45

Xmax	Mean
82	50
100	54
115	58
130	62
143	67
155	70
166	74

Median: 60

Xmax	Mean
96	64
114	68
132	72
148	76
162	80
176	85
186	89

Median: 90

Xmax	Mean
144	96
162	100
181	104
198	108
214	112
229	116

Median: 120

Xmax	Mean
155	122
173	125
192	128
211	132
229	136

Median: 180

Xmax	Mean
232	184
250	186
269	189
288	192
307	196
326	199

making a partial extension at both ends of these "Median curves" (according to how we believe it would go if we had more data points). The reader should convince himself that this was justified due to the pattern indicated by the given data points.

Furthermore, to give a more realistic picture, we have used a different unit on the vertical scale than we have on the horizontal scale. Our choice was dictated by the fact that a given change in the Mean value is more readily noticed than the same change in Xmax. The ratio we used was 1 to 2--although some might argue with this particular choice, we feel certain that all would agree that the unit on the horizontal axis should be larger than the unit on the vertical axis.

Now let us see what these Median curves tell us. Let us first analyze what happens as the Xmax value changes with the Mean value held fixed. Since any vertical line represents a fixed mean, we see that moving upward on a vertical line corresponds to increasing the Xmax value for any given Mean. Figure 2, below, has enlarged two of these Median curves and such a vertical line has been superimposed.

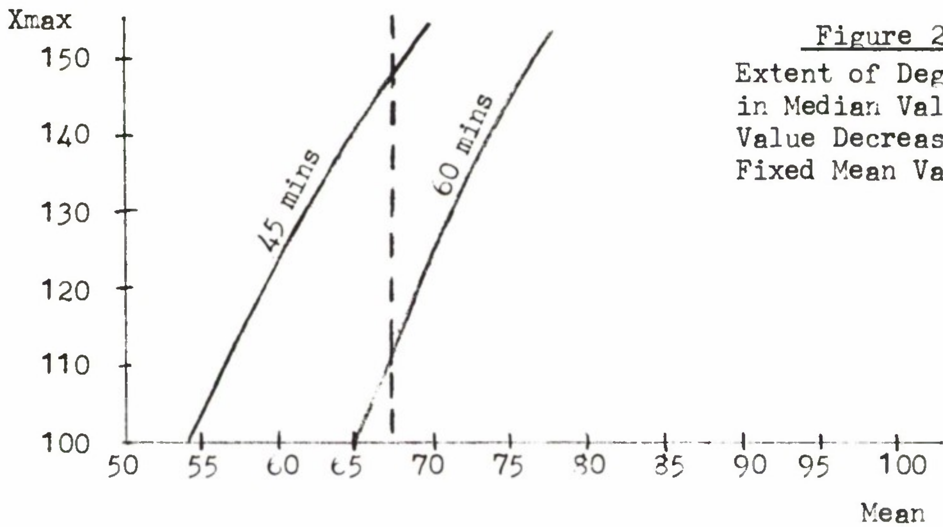


Figure 2

Extent of Degradation in Median Value as Xmax Value Decreases for Fixed Mean Value

Obviously, most vertical lines would intersect two or more Median curves (in fact, all vertical lines would do so, if we had been able to show more Median curves). The important fact, however, is that we intersect a larger-valued Median curve first. We therefore conclude that as the Xmax value increases, an undesirable change, the Median value decreases, a desirable change! Moreover, as we make small decreases in the value of Xmax, we induce a relatively large increase in the Median value.

We shall now paraphrase these results in Maintainability language. For the Lognormal class, the following property holds:

"For a fixed Mean, to slightly improve (lower) that value for which 90% of all repairs will not exceed, we must tolerate a (relatively) large increase in the value for which 50% of all repairs will not exceed. Thus, we cannot lower (improve) one of these without raising (adversely affecting) the other, unless we also lower the Mean value."

Whether or not this is realistic is another matter--let us first discuss the consequences of this special property.

A discerning reader should be saying to himself: "Yes, I agree, but if only a slight reduction in the Mean value is required to leave the Median value virtually untouched, then this property would be of no practical importance." Also, those who are statistically oriented might be thinking: "This is a natural consequence of lowering (raising) the Standard Deviation which, in essence, is being done when we lower (raise) the Xmax value--and, surely, we can find ways to compensate for this property."

Let us try to answer both propositions at once. We would like to give prospective contractors adequate guidance--by adequate we mean simply this: he should be able to take our quantitative requirements and immediately know what constitutes an improvement over these requirements. Obviously, if we specify the Mean and Xmax values, we are in effect telling him that he need not devote his efforts to making repair actions take as little time as possible. In fact, he would be hurting himself if he did, particularly if we used the "sample standard deviation[1]" to demonstrate compliance with the Mean value. More specifically, we would be telling him to effect changes to those repair actions which depart too much--in either direction--from the Mean value! That is to say, raise those which are too short and lower those which are too long. Ridiculous? Let us hasten to add that we think so too.

The following example shall be used to indicate how large a change could take place in the vicinity of "short repair-times" in an effort to meet our requirements. Let us suppose that a Mean of 1 hour and Xmax of $2\frac{1}{2}$ hours were set forth. These requirements would be satisfied by either Curve A or Curve B below. These density functions have been taken from the Appendix. Both have the same Mean value (58 minutes) and both have lower Xmax values than the required $2\frac{1}{2}$ hours.

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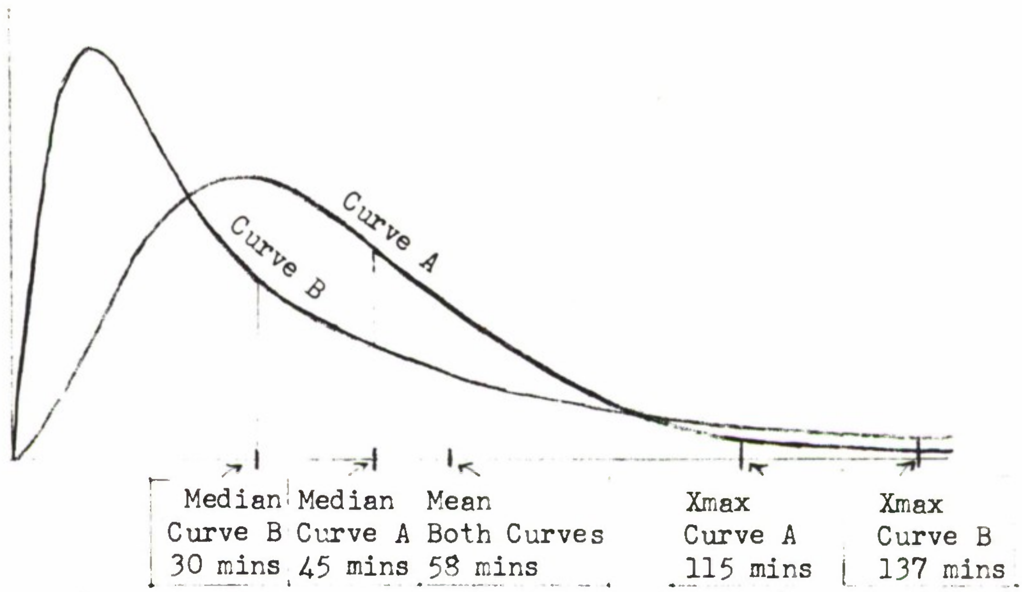
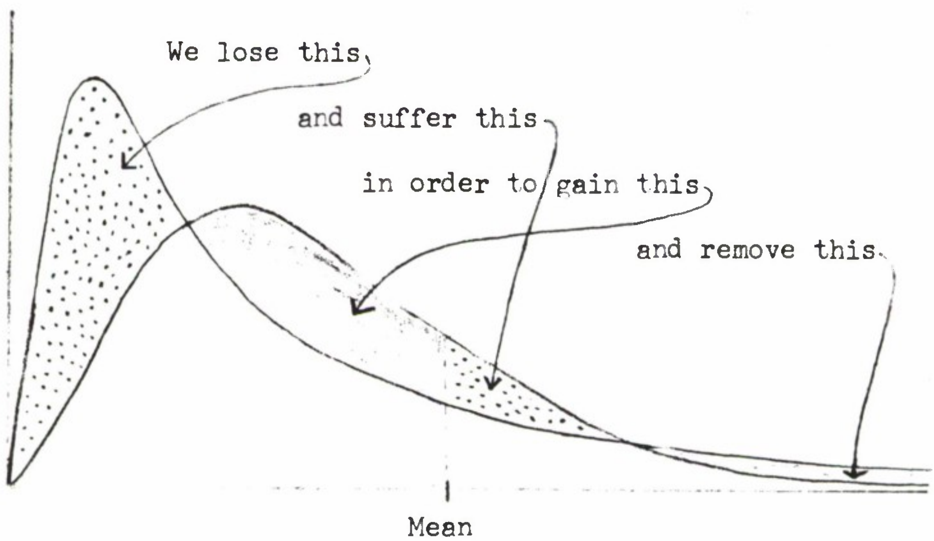
[1]The "sample standard deviation" will be explained when we discuss demonstration methods.



However, the contractor would stand a better chance of convincing us he has met requirements if he designed for Curve B instead of Curve A. The consequences of his doing so are illustrated below:



It should be observed that, in this case, a small <u>decrease</u> (10%) in the Xmax value has brought about a large <u>increase</u> (50%) in the Median value. Which of these curves indicates a better Maintainability design? We feel that most would agree that for certain applications Curve B would be <u>needed</u>, whereas for other applications Curve A would be more <u>economical</u>, depending on how seriously long repair times would affect mission achievement.

It must be remembered that we are merely discussing mathematical properties of a given class of functions (in spite of the fact that they have a severe impact on our specification and verification of numerical requirements). In a real situation, there may only be two or three repair actions which cause the achieved value of Xmax to be greater or lower than desired. These could be corrected during design reviews, thereby giving us a density curve that is similar to the left hand portion of Curve B and the right hand portion of Curve A. That is, our achieved values would be:

Xmax	Mean	Median
115	58	15

which represents an improvement over both curves. These values, of course, do not correspond to a lognormal density; however, we have been trying, in this report, to convince those responsible for developing Maintainability requirements that we must reject the lognormal function as a design criterion--its usefulness in this respect is extremely limited!

We have still not answered the question: how much does the Mean value requirement (60 minutes) have to be lowered in order to leave the Median value (15 minutes) intact, as we lower the Xmax value from 137 minutes to 115 minutes? The curves in Figure 1 provide the answer; namely, 10 minutes, that is, about 17%. In other words, a 17% reduction in the Mean value is required (to leave the Median value unchanged) when making a 10% reduction in the Xmax value. This, to us, represents a large reduction, relatively speaking, of course.

Naturally, our guidance would be more effective if we gave all three; but realizing that people are "creatures of habit", we shall now show that if only two of these are specified that the Median and Xmax is a better choice. This can be seen quite readily by examining the slopes of the curves in Figure 1. Moving along any particular Median curve corresponds to holding the Median value fixed--apparently, "large" changes in the Xmax value are required to bring about "small" changes in the Mean value, when the Median is held fixed. Moreover, the Mean decreases as Xmax decreases (and vice versa) just as we would like to happen. In other words, an improvement in one necessarily brings about an improvement in the other. In this case, there is no question as to how to improve upon the stated requirements--minimize both as much as possible!

There is only one remaining argument (that we can foresee) against our recommendation of specifying (and subsequently demonstrating) the Median value and the Xmax value. It is this: Operational requirements are often stated in terms of Availability which is most often defined as

$$\frac{\text{(Mean-time-between-failure)}}{\text{(Mean-time-between-failure)} + \text{(Mean-downtime)}}$$

that is, we often know the <u>Mean</u> value requirement, but not the <u>Median</u> value. Although this does not alter the previous considerations at all, we feel it might be helpful to tabulate various Median value possibilities when (typical) values of the Mean and Xmax are given. Table 2 (next page) does just that, and may be read the other way also; that is, having demonstrated particular Median and Xmax values, that are shown in the Table, one can derive a Mean value that corresponds to them. Evidently, Figure 1 can be used for this same purpose.

TABLE 2

MEDIAN VALUES CORRESPONDING TO MEAN AND Xmax VALUES

Xmax \ Mean	30	45	60	90	120	180	
50	25						
60	20						
75	16	40					
90		33	55				
105		20	50				
120			47				
135				83			
150				75			
165				70			
180				64	110		
195					105		
210					100		
225					95		
240						155	
270						150	
300						140	
360						120	

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2. Mathematical Considerations:

This section gives specific mathematical details of the lognormal class - its derivation and special formulae relating certain parametric values. It begins at a relatively low level, but rapidly rises to a high peak; if the reader finds that the slope is too steep, he should skip to Chapter IV, most of which does not depend on this section.

We have tried to restrict the discussion to those aspects which directly relate to the decision problem, but occasionally a digression is necessary in order to obliterate certain false notions that seem to be held by too many people. Before we begin, a few words must be said about our choice of functions and notation. We use the natural logarithm function only, denoted by "ln t", because of its simplicity. However, since $(\ln a)(\log_a t) = \ln t$ (i.e., the natural logarithm of t is a constant multiple of "the logarithm of t with respect to another base a"), everything we say in the sequel would basically be true for any other base - only the numerical examples (parameter values), used to illustrate the ideas, would change.

a. Definition. Although the lognormal distribution has often been defined as "the distribution of a random variable whose logarithm obeys a normal law of probability," this definition is actually misleading. Since the expression

$$\boxed{g(x) = c\, e^{-\frac{1}{2\rho^2}(x-\omega)^2}}$$

where $c = 1/\rho\sqrt{2\pi}$, defines a normal density function, users of the theory might be led into the trap of assuming that the lognormal density function $f(t)$ is defined simply by replacing x by ln t in the right side of the above equation. This is incorrect - instead, we must proceed by examining the cumulative distribution function. In these terms, we are given that

$$G_x(a) = \int_{-\infty}^{a} g(x)\, dx = P\{x \leq a\}$$

(with $g(x)$ defined as above) is the cumulative distribution function for a random variable (x) which is obtained from another random variable (t) by the transformation $x = \ln t$. In order to determine the distribution of the "untransformed variable" we use the "change of variable" technique of elementary calculus, which then causes the limits of integration to change as follows:

$$G_x(a) = G_{\ln t}(a) = \int_0^{e^a} g(\ln t)\ d(\ln t)$$

Letting $t' = e^a$, and since $d(\ln t) = \frac{1}{t}\,dt$,

$$G_x(a) = \int_0^{t'} \frac{1}{t}\ g(\ln t)\ dt$$

$$= F(t') \qquad \text{(Say)}$$

One easily verifies that F satisfies all the properties that a cumulative distribution function is expected to have (since G_x does); in fact, since the logarithm function is non-decreasing, we know

$$\begin{aligned} P\{t \leq t'\} &= P\{\ln t \leq \ln t'\} \\ &= P\{x \leq \ln t'\} \\ &= G_x(\ln t') \\ &= F(t') \qquad \text{(Since } e^{\ln t'} = t'\text{)} \end{aligned}$$

that is, $F(t') = P\{t \leq t'\}$. Now,

$$F(t') = \int_0^{t'} \frac{1}{t}\ c\ e^{-\frac{1}{2\rho^2}(\ln t - \omega)^2}\ dt$$

and since $c = \frac{1}{\rho\sqrt{2\pi}}$, we have finally verified that the lognormal density function, f, must be defined by the rule:

$$\boxed{f(t) = \frac{1}{t\rho\sqrt{2\pi}}\ e^{-\frac{1}{2\rho^2}(\ln t - \omega)^2}}$$

b. Relations Between g(x) and f(t). It is important[1] to note that the mean and the variance of the untransformed variable (t) is not ω and ρ^2; these are the mean and variance of the transformed variable (x).

In other words, the location and shaping parameters of the untransformed variable have not yet been given. They are found by manipulation in accordance with the definitions given in Chapter I, and appear below:

The Mean: $M(t) = \mu = e^{\omega + (\rho^2/2)}$

The Median: $M^*(t) = \theta = e^{\omega}$

The Mode: $Mo(t) = \xi = e^{\omega - \rho^2}$

The Variance: $V(t) = \sigma^2 = \left(e^{2\omega + \rho^2}\right)\left(e^{\rho^2} - 1\right)$

A glance at these formulae reveals some important properties of the log-normal distribution. First, since the median $\theta = e^{\omega}$, taking logarithms of both sides gives us:

$$M(x) = \omega = \ln \theta$$

In words,

The logarithm of the median of the untransformed variable is equal to the mean of the transformed variable.

Second, these formulae show that no other parameters, thus far stated, have this simple relationship! In particular, we wish to emphasize that

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The mean of the transformed variable is not a function of any single parameter of the untransformed variable.

We shall find that this knowledge is indispensable when we try to develop a decision rule for Maintainability demonstration.

[1] To keep things straight, with a minimum number of words, we shall always call the random variable described by f(t) the "untransformed variable," and that described by g(x) the "transformed variable."

The formulae for μ and σ^2, given above, may be solved simultaneously to derive formulae for ω and ρ^2 in terms of μ and σ^2, which we now state:

$$M\{x\} = \omega = -\frac{1}{2}\ln\left(\frac{\mu^2+\sigma^2}{\mu^4}\right)$$

and once ω is known we have

$$V\{x\} = \rho^2 = (2\ln\mu) - (2\omega)$$

We may verbalize these facts as follows

The mean and variance of the transformed variable are functions of <u>two</u> variables; i.e., each is a function of the <u>mean and variance</u> of the untransformed variable.

Chapter IV of this document explains the practical significance of these underlying relationships, which comes to the surface as we attempt to analyze repair-time data.

CHAPTER IV

RECOMMENDED DEMONSTRATION METHODS

1. The Meaning of Demonstration

Repair-time data is analyzed for three different purposes. It is our belief that many people have not given careful thought to distinguishing between the three, which causes problems when they try to communicate with each other. Therefore, we hope to eliminate this confusion before attempting to give our demonstration methods. However, the reader should not think of the following discussion as a digression; we have only one of these purposes in mind--if he has another, he may gain very little from learning our methods. Furthermore, we will introduce certain terminology that will be used in explaining our demonstration methods.

The three purposes are:

(i) Prediction

(ii) Estimation

(iii) Verification (Decision-Making)

We shall learn that one reason for the confusion is that similar methods may sometimes be used to accomplish these three different objectives. For the most part, however, the methods are quite different.

Prediction aims at finding the specific density function that fits the equipment or system in question. (We leave to the reader to fill in why this knowledge is sought for prediction purposes--the "why" of it is not pertinent to the present aim of comparing "how" these objectives are accomplished.) To accomplish a prediction exercise, quite a bit of data must be collected, particularly if there is no "a priori" information concerning the sought-after function. There are no specific rules that dictate "how much" before the data collection begins. However, if the class of functions that governs the repair-time behavior is already known (for example, if it is known that the Lognormal class applies) then the goal of prediction is to get a "best estimate" (sometimes called "point estimates") of the parameters of that class of functions. Depending on the class involved, there may or may not be specific rules which dictate how much data to collect to obtain such estimates. Occasionally, certain "estimation" techniques are used to attain the desired goal, but this does not alter the fact that the main purpose is to find the specific density function.

Estimation aims at gaining information about the parameters of a given (sometimes not given) class of functions. It is usually not concerned with finding the density function itself--a point that cannot be emphasized too strongly. There are two general methods used for estimation purposes. When very little is known about the class involved (only that such functions are continuous, for example) the so-called "non-parametric" technique is usually used. This technique (whose name may be misleading) tries to obtain estimates concerning certain parameters without directly depending on knowledge of a particular class of functions which governs the basic random variable. The name "distribution-free" has been suggested as a substitute for "non-parametric," and this is a better choice for obvious reasons.

Another method for estimation has been given the name "confidence interval estimation." This method rests quite heavily on knowledge of the particular class of functions involved, and uses this knowledge directly to obtain estimates of parameters. Quite often, "confidence interval" techniques are used to gain information concerning the "average value" since, as we have previously pointed out, this kind of knowledge is essential to give the concept of "average" precise meaning. Naturally, one would not use a certain class to give meaning to "average" and then use a non-parametric approach to get estimates of the average value which, in effect, says "I'm not quite sure what class of functions applies to this random variable."

The output of the two types of estimation methods is pretty much the same--a probability statement (usually called a "confidence interval" statement) that the parameter in question falls within a certain "spread" of values. Depending on the amount of data collected this "spread of values" (called an "interval") may be relatively large; whence it may convey little useful information for making decisions. How large a sample must be collected to yield interval estimates of a certain <u>restricted</u> size, at a given level of probability, may <u>sometimes</u> be predetermined. In many cases, however, this is not possible; that is, the size of the interval can only be determined <u>after</u> the data has been collected. This situation usually occurs when the given class of function has two parameters both of which are unknown; for example, the Lognormal class.

The above discussion of <u>prediction</u> and <u>estimation</u>, brief as it was, should have been sufficient to point out that neither is an appropriate tool for decision-making. The methods used are <u>particularly</u> inadequate for making decisions concerning compliance or non-compliance with <u>contractual</u> requirements--which is the type of decision-making (verification) that this report is mostly aimed at. Most decision-making requires that the amount of data that will be collected be stipulated in advance; that is, <u>before</u> data collection begins. Moreover, an unambiguous rule must be established which tells us which decision to make--accept or reject--on the basis of the collected data. Such decisions always involve a certain degree of "risk" to both parties (Air Force and Contractor); that is to say, these evaluations cannot be made with certainty. Therefore, it is important to realize that the "risks" must be <u>quantified</u> in order that they may be controlled. This quantification of risks is easily accomplished when the class of functions that pertains to the random variable under consideration is known.

It is not our intent to present decision theory for its own sake--only to show how we apply certain techniques to solve the Maintainability demonstration problem. We shall give further elaboration of the decision-making approach as we give the details of our demonstration method.

2. Practical Application of Prediction, Estimation, and Demonstration Methods[1]

We hope that these introductory remarks have caused the reader to see the basic differences between prediction, estimation, and decision-making--the latter, of course, being equivalent to "demonstration." Let us give one practical example, however, to illustrate these differences.

Suppose we require a vehicle-mounted communications system composed primarily of UHF, VHF, and HF radio sets. Suppose mission requirements call for an Availability figure of 90%, and that a "point estimate" MTBF figure of 180 hours has already been obtained. Examining the ratio

$$\frac{\text{MTBF}}{\text{MTBF} + \text{MDT}} = \frac{180}{180 + \text{MDT}} = .996$$

tells us that our mean-down-time (MDT) figure must be 44 minutes or less. A prediction exercise would now be conducted by the Air Force to establish quantitative requirements (and subsequently by prospective contractors to determine whether they can meet the requirements). For example, using empirical data, the Air Force may decide that, in the past, the Lognormal has applied to this type of gear, in which case they would see (from the curves contained in the Appendix, or the Median curves in Figure 1) that a Median value of 30 minutes and an Xmax value of 97 minutes corresponds to a Mean value of 44 minutes. Estimation techniques (using field data) may be used, as well, to determine the reasonableness of these quantitative requirements. For example, if one has (say) one hundred samples of repair times, labeled

$$t_1, t_2, t_3, \ldots\ldots\ldots\ldots, t_{100}$$

and if one takes (natural) logarithms of these values and labels them

$$x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_{100} \qquad (x_i = \ln t_i)$$

then the computation of

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots\ldots\ldots + x_{100}}{100}$$

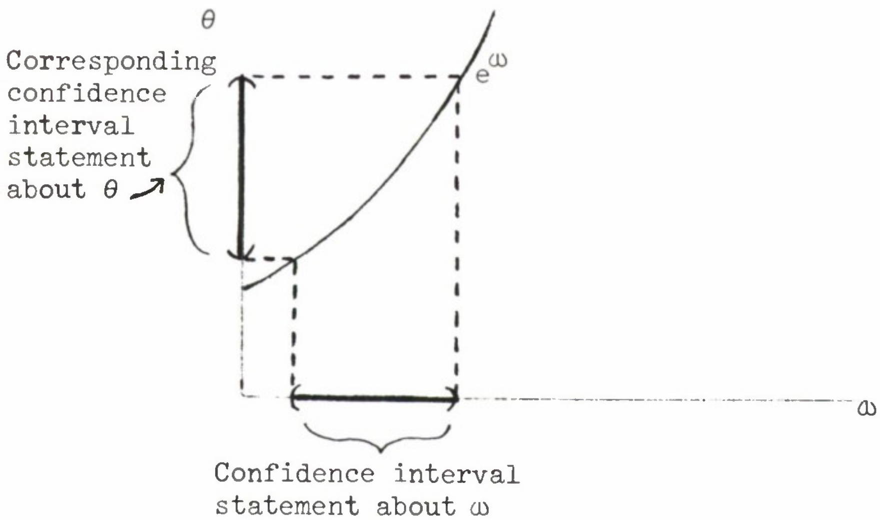
gives a (best) estimate of the Mean of the <u>transformed</u> variable (x). Knowing that the transformed variable obeys a Normal distribution with Mean value ω (say), one may use students-t distribution to get a confidence interval estimate of ω. Now since

$$\omega = e^{\theta}$$

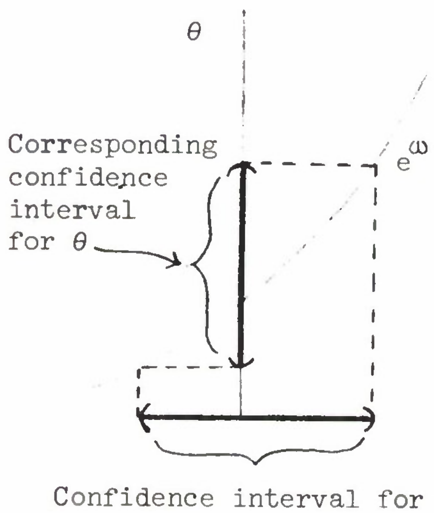
[1] This section rests quite heavily on terminology and results developed in Section 2 of Chapter III. Those who have not mastered that section should skip over to Section 3.

is monotonic increasing, where θ is the Median value of the untransformed variable, the confidence interval estimate of ω may be translated (in one-to-one fashion) to a confidence interval statement about the Median value θ. The following examples show this quite clearly:

Example 1



Example 2



We note, in passing, that methods for finding "exact" confidence intervals for the Mean value (of the basic random variable) are yet to be developed. For those who are interested in gaining further insight into estimation techniques we urge them to consult "The Lognormal Distribution" by Aitchison and Brown published by The Sydics of the Cambridge University Press (1957).

Hence, it is rather easy for both parties (Air Force and Contractor) to obtain confidence interval estimates of the Median value (or the Mean value of the <u>transformed</u> variable), thereby providing further justification for giving the Median value explicit mention.

It should be added that if abundant repair-time data is available (a rare occurrence) one could treat the "sample Mean" or the "sample Standard Deviation", defined by:

$$\bar{t} = (1/n)\sum_{i=r}^{n} t_i \qquad \text{(sample mean)}$$

$$\bar{s} = (1/n)\sum_{i=1}^{n} (t_i - \bar{t})^2 \qquad \text{(sample standard deviation)}$$

as obeying a normal distribution. Besides the fact that there are no rules for determining how large n must be for this technique to be appropriate, one must be rather careful when analyzing $\bar{s}$; that is, as previously mentioned, higher values of $\bar{s}$ give rise to "better" Median values (more generally, the left hand portion of the density curve is <u>improved</u>), but they also produce "worse" Xmax values (more generally, the right hand portion of the density curve is <u>degraded</u>). Thus, one must be on guard not to become so enamored with his "objective" statistical approach that he loses sight of how to interpret his results. This lack of "order" concerning the "sample Standard Deviation" makes it a poor measure to use for the <u>demonstration</u> of "percentile requirements." It is only useful when coupled with investigations of the Mean value; for example, if one seeks "point estimates" of the parameters of the density function for prediction purposes. In this regard, another theorem that might be useful for gaining point estimates is: (see Section 2 of Chapter III for definitions of symbols)

$$P\left\{z \leq \rho/2\right\} = \int_0^{\mu} f(t)\, dt = \int_{-\infty}^{\frac{\rho}{2}} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2}\, dx$$

In words, the area to the left of the Mean of the <u>untransformed</u> variable is a function of the standard deviation (alone) of the <u>transformed</u> variable. Rephrased in Maintainability language, this theorem tells us that the probability of completing a repair in less time than the Mean value is equal to the area to the left of the point $\rho/2$ in a standard normal distribution (Mean zero, and Standard Deviation 1), where ρ is the standard deviation of the transformed variable. Thus, one could set "the percentage or sample repair-times that take less than the sample Mean $\bar{t}$" equal to the last integral on the right, above, and use normal tables to solve for ρ . If one combines an interval or point estimate of ω with this point estimate of ρ , one then has a rough idea of the density function (assuming lognormality, of course).

Now that we have seen how "estimation" techniques are used, we would like to emphasize that it would be a serious mistake to use such techniques for decision-making purposes. To see why this is so, let us suppose that a 90% confidence level is chosen, and the demonstration (data) tells us that our interval is fifteen minutes to one hour; that is, we have 90% confidence that the Median-time-to-repair lies somewhere between those two limits. (Recall that our requirement was for one-half hour.) How do we make a decision? Do we use the lower limit, fifteen minutes, or the upper limit, one hour, to make a decision? Evidently, if we required the upper limit to fall below one-half hour (before we would accept), this would be too strict, but if we required the lower limit to fall above one-half hour (before rejection), this would be too lenient. Anyway, either stipulation leads to no decision in most cases. This example should make it apparent that estimation techniques are not designed to make decisions. Another approach is needed.

Having shown how prediction and estimation techniques are applied to this practical example, it remains to discuss demonstration methods. This shall be covered in detail in subsequent sections. Here, we simply state that our decision-making approach will be "distribution-free" in nature, which simply means that we will try to develop a decision rule that does not depend upon the Lognormal class. The reasons for this have already been covered in detail and will not be repeated.

3. Rationale for the Demonstration Method

We recommend that Wald's Sequential Analysis[1] technique be used to develop what we shall call a "Sequential Test of Percentiles." The specific test procedure shall be given in the next section. In this section we shall simply record our reasons for choosing Sequential Analysis techniques, without giving further elaboration of these ideas. In stating these reasons we may have used certain statistical and/or Systems Command "jargon" which are not defined elsewhere in this report. We ask the reader to excuse us on the grounds that this report may already be "too long" for its content.

Our reasons are:

a. Fewer observations (maintenance actions) are permitted, for a given level of confidence, than the "fixed" sample size approach. This results in considerable savings in "test dollars" without affecting our assurance that requirements have been met.

b. Air Force and Contractor risks may be explicitly quantified and evaluated. Thus, "the probability of accepting unsatisfactory equipment" (usually called consumer risk) and "the probability of rejecting satisfactory equipment" (usually called producer risk) are carefully controlled.

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[1] See Wald's "Sequential Analysis," John Wiley & Sons (1947).

c. Methods of sequential analysis are better suited to tests conducted under operational conditions (e.g., Category II testing) where failures are not simulated. In such cases, the number of expected maintenance actions is unknown and dependent upon the failure-rate of the system/equipment.

d. When all (or most) testing must be conducted in an operational environment, R&M testing and evaluation may be conducted simultaneously, using a similar criterion for decision-making, thereby reducing the total cost of testing and encouraging trade-off analyses or, at least, providing an awareness of the effect that one of these system parameters has upon the other.

4. Sequential Test of Percentiles

We assume now that our quantitative requirements shall chiefly be stated as

a. The Median-time-to-repair (MTTR), i.e., the 50th Percentile.

b. Xmax, i.e., the 90th Percentile.

Thus, we require a technique that tests these particular percentiles of the distribution. We shall see that this is quite easy due to the direct connection between "percentiles" and "probability" (the cumulative distribution function).

By now, it should be clear that if the true value of the Median is equal to the specified value, then the probability that a repair action will take less time than the specified value is 50%. Consequently, having observed say 50 repair actions, if only 10 of these take less time than the specified value and the remaining 40 take more time, we would seriously question whether the true Median is within reasonable limits of the specified value. On the other hand, if 40 fall below and only 10 fall above, we would be conducive to the belief that the true Median is at least as good as the specified value.

It should also be clear that our problem reduces to how many must fall below the specified value to cause immediate acceptance, and how many must fall above the specified value to cause immediate rejection. For example, if 25 fall above and 25 fall below, we may still entertain doubts as to what would happen if we took 50 more samples of repair-times. Similar remarks apply to the Xmax value; that is, we would have difficulty making a decision if 45 fell below Xmax and 5 fell above, since this is exactly 90%.

Fortunately, mathematicians have been thinking about this type of problem for some time, and have derived for us what is called the binomial frequency function which applies to experiments which are only interested in the occurrence or non-occurrence of an event. If we know the probability p of the event occurring in any one trial of the experiment, so that the probability of the event not occurring is (1-p), then the probability of obtaining exactly X occurrences of the event in n trials defines what is known as the binomial frequency function. This function is denoted by b(n;x;p) because it depends on n, x, and p, where:

n = the number of trials to be made

x = the total number of occurrences of the event in question

p = the probability of the event occurring in any one trial.

It is fairly easy to prove that this frequency function takes the form

$$b(n;x;p) = \frac{n!}{x!\,(n-x)!} \cdot p^x \cdot (1-p)^{n-x}$$

$$= \left\{ \begin{array}{l} \text{The probability of obtaining X occurrences of} \\ \text{an event in n trials if the probability of the} \\ \text{event occurring in one trial is p.} \end{array} \right\}$$

so that it will not be proven here[1]. This function will be quite useful in setting up a decision rule to test our percentile requirements. Specifically, it represents a mathematical model, not of the "time-to-repair" statistic, but of "the number of repairs that take less (or more) time than any given percentile." Accordingly, it may be used to quantify and therefore control the risks involved in making accept-reject decisions.

A nice feature of the binomial frequency function is that it does not depend upon the Lognormal class at all. In fact, the binomial applies to percentiles no matter what class of functions governs the "time-to-repair" statistic.

In order to keep this report within reasonable proportions, we cannot go into detail concerning sequential analysis techniques. This is done in Section V of ESD TDR-64-616, "Handbook for Reliability and Maintainability Monitors." Although that section concerns itself mostly with Reliability decision-making, Chapter 3 of that report deals with the decision problem in general. For this report we shall be satisfied to give an accept-reject criteria, accompanied with a brief explanation, that would be useful for making decisions concerning the specified Median and Xmax values. Appendix B gives the mathematical equations used to develop Table 3.

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[1] The proof may be found in any standard text on probability theory.

ACCEPT-REJECT CRITERIA
SEQUENTIAL TEST OF PERCENTILES
TABLE 3

Total Number of Tasks

θ = 50th Percentile

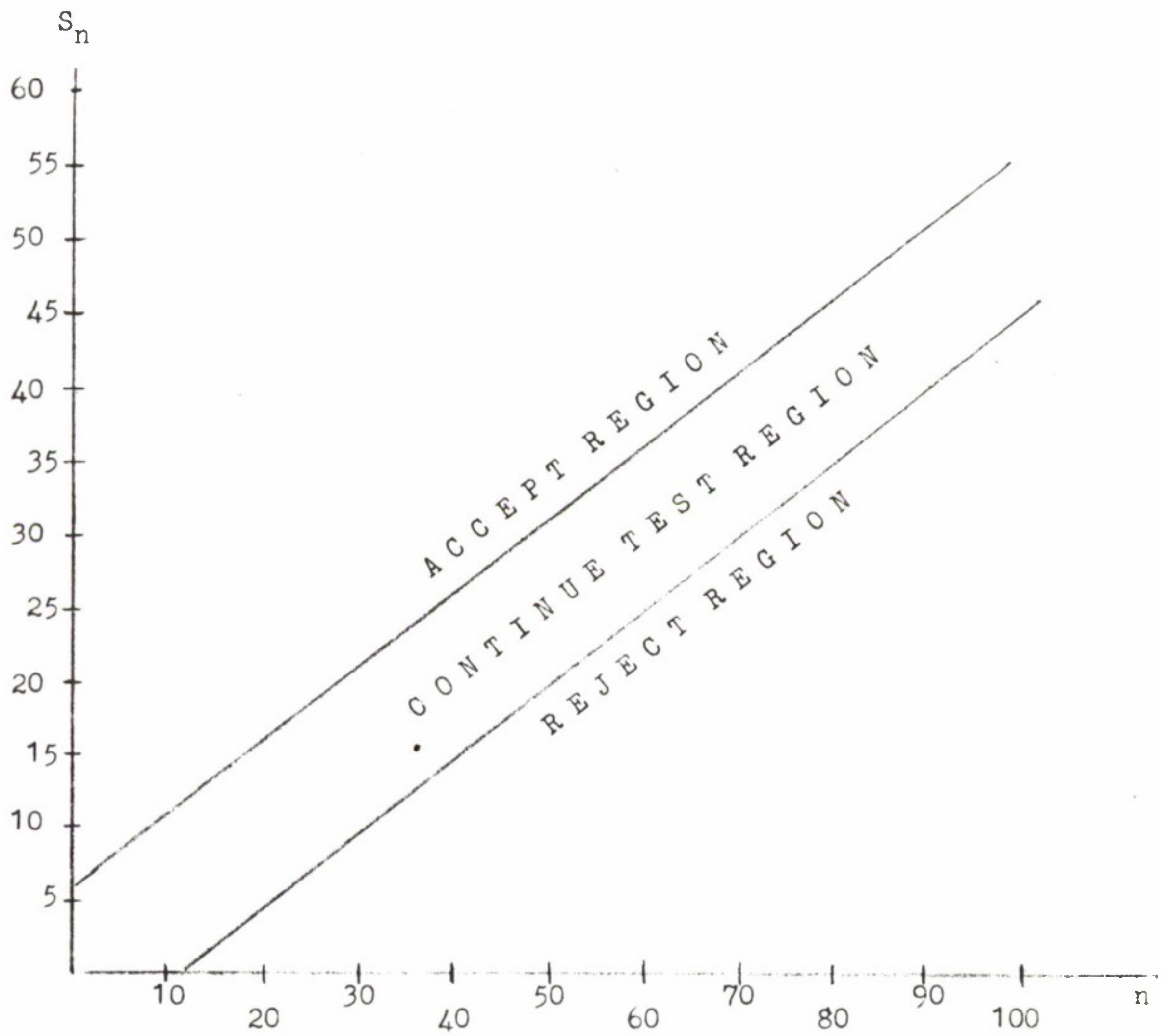
S_n = {Number of tasks that take less time than θ}

Xmax = 90th Percentile

S_n^* = {Number of tasks that take less time than Xmax}

N	ACCEPT (θ = 50th Percentile)	REJECT (θ = 50th Percentile)	CONTINUE (θ = 50th Percentile)	ACCEPT (Xmax = 90th Percentile)	REJECT (Xmax = 90th Percentile)	CONTINUE (Xmax = 90th Percentile)
50	31	19	20-30	--	40	40
51	31	20	21-30	--	41	41
52	32	20	21-31	--	42	42
53	32	21	22-31	--	43	43
54	33	21	22-32	--	44	44
55	33	22	23-32	55	45	46-54
56	34	22	23-33	56	46	47-55
57	34	23	24-33	57	46	47-56
58	35	23	24-34	58	47	48-57
59	35	24	25-34	59	48	49-58
60	36	24	25-35	60	49	50-59
61	36	25	26-35	61	50	51-60
62	37	25	26-36	62	51	52-61
63	37	26	27-36	63	52	53-62
64	38	26	27-37	64	53	54-63
65	38	27	28-37	65	54	55-64
66	39	27	28-38	65	55	56-64
67	39	28	29-38	66	56	57-65
68	40	28	29-39	67	56	57-66
69	40	29	30-39	68	57	58-67
70	41	29	30-40	69	58	59-68
71	41	30	31-40	70	59	60-69
72	42	30	31-41	71	60	61-70
73	42	31	32-41	72	61	62-71
74	43	31	32-42	73	62	63-72
75	43	32	33-42	74	63	64-73
76	44	32	33-43	75	64	65-74
77	44	33	34-43	75	65	66-74
78	45	33	34-44	76	66	67-75
79	45	34	35-44	77	67	68-76
80	46	34	35-45	78	67	68-77
81	46	35	36-45	79	68	69-78
82	47	35	36-46	80	69	70-79
83	47	36	37-46	81	70	71-80
84	48	36	37-47	82	71	72-81
85	48	37	38-47	83	72	73-82
86	49	37	38-48	84	73	74-83
87	49	38	39-48	85	74	75-84
88	50	38	39-49	85	75	76-84
89	50	39	40-49	86	76	77-85
90	51	39	40-50	87	77	78-86
91	51	40	41-50	88	77	78-87
92	52	40	41-51	89	78	79-88
93	52	41	42-51	90	79	80-89
94	53	41	42-52	91	80	81-90
95	53	42	43-52	92	81	82-91
96	54	42	43-53	93	82	83-92
97	54	43	44-53	94	83	84-93
98	55	43	44-54	95	84	85-94
99	55	44	45-54	95	85	86-94
100	56	44	45-55	96	86	87-95

A graphic representation of the 50th Percentile decision rule is given in Figure 3.



SEQUENTIAL TEST OF 50TH PERCENTILE

FIGURE 3

A similar graph can be plotted for the 90th Percentile test by consulting the equations given in Appendix B.

Let us briefly explain the use of Table 3. No decision is made until 50 repair actions have been completed. At the completion of 50 tasks we begin by computing

S_n = Total number of tasks that take less time than the contractual Median-time-to-repair (MTTR), θ, in n trials.

S^*_n = Total number of tasks that take less time than the contractual Xmax, in n trials.

For example, if $S_{50} = 31$, we accept. However, if $S_{50} = 19$ or if $S^*_{50} = 40$, we would reject. As soon as one of S_n or S^*_n causes an accept decision, we stop computing that statistic and continue computing the other until reaching an accept decision for that one also. If either of these statistics causes a reject decision, our decision is to reject. If both statistics remain in the continue test region throughout the test (through 100 tasks) we would then make an accept decision.

This accept-reject criteria makes decisions with a maximum of 10% risks for both parties by making the assumptions given in Appendix B. There, it is indicated that both parties are required to depart a small (equal) amount from the specified values in order to compute their (maximum) risks. Furthermore, such departures are necessary to give relatively low and equal risks to both parties throughout the test.

It may appear that the Air Force incurs a higher degree of risk if we remain in the "continue test region" throughout the test, since we have stated that an accept decision is made when this occurs. Actually, this is not so for the following reason: The longer we stay in the "continue test region," the more assurance is gained that the specified MTTR value, θ, is between the 45th and 55th Percentile, and the specified Xmax value is between the 88th and 92nd Percentile. Although, no-one has taken the trouble to quantify this added assurance, it is nonetheless _intuitively_ clear that this is so. Section V of ESD-TDR-64-616 gives a suggestion for how such a quantification could be carried out, if such was needed. For our purposes, it does not appear to be necessary.

5. Selection of Maintenance Tasks

This study did not investigate methods for selecting maintenance tasks to be simulated (in-plant or elsewhere), although thoughts about this aspect nagged us often, and some might hold to the belief that this is a more severe problem than the one we tackled. This does not mean that we draw a blank in this area; we do have some thoughts which we shall now state.

First and foremost, we do _not_ believe that this problem can be completely solved by stating a given _generalized_ procedure for the selection of maintenance tasks (at least not for electronic equipment which are composed of thousands of parts). _Ideally_, any procedure should take into account:

a. Failure rates, so that units which fail most often are given a large share of the total number of tasks to be simulated.

b. Repair rates, so that units which are responsible for the major portion of the total down-time (for any given period) are given an appropriate share of the number of tasks to be simulated.

But in a practical case, it may not be feasible to use b, above, as part of the procedure. The reason for this is clear: if we allow (gross) estimates of repair rates to influence the task selection, and if such estimates are wrong, we run the risk of biasing our results. If we could assume that the contractor could provide good estimates in this area, or that we could evaluate such estimates, there would be no need to demonstrate in the first place.

One could say that these same comments apply as well to estimates of failure rates; but there is a significant difference. Failure rates are analyzed (and demonstrated) separately--often by different people. Also, it is easier to detect (gross) errors in failure rate estimates (which are bad enough to create bias) due to what we call the "experience factor."

In any case, the contractor should be given a definite procedure for the selection of maintenance tasks from which he may depart only if approval is obtained from the procuring activity. Needless to say, close scrutiny by the procuring activity is required. For the reasons mentioned above, the following procedure is the best that we have seen to date; however, it is not appropriate for verbatim-application to contractual documents. It merely represents a bare framework which must be given further elaboration in any individual procurement situation (in light of the design characteristics of the equipment involved). Such words as "unit", "group", etc., must be defined in order that the number of tasks to be demonstrated are not spread "too thinly" over the myriad collection of parts, thereby giving an unrealistic demonstration effort.

The procedure is as follows:

a. Obtain the failure rate of each unit (the sources used must be approved by the procuring activity).

b. Group together similar units or homogeneous classifications into r groups, and for each group i ($i = 1, 2, \ldots, r$) determine g_i, the average group failure rate.

c. For each group i, determine h_i, the number of items in the group.

d. Apportion a minimum of n maintenance tasks to the r groups in proportion to their respective failure rate averages and the number of items in each group in accordance with the following formula:

$$x_i = \frac{(g_i)(h_i)n}{\sum_{i=1}^{r} (g_i)(h_i)}$$

where x_i = the number of tasks to be simulated in a particular group i. NOTE: If x_i is not an integer, the least positive integer greater than x_i will be used.

e. The x_i tasks shall be apportioned to the particular group items by the use of a random number table.

APPENDIX A

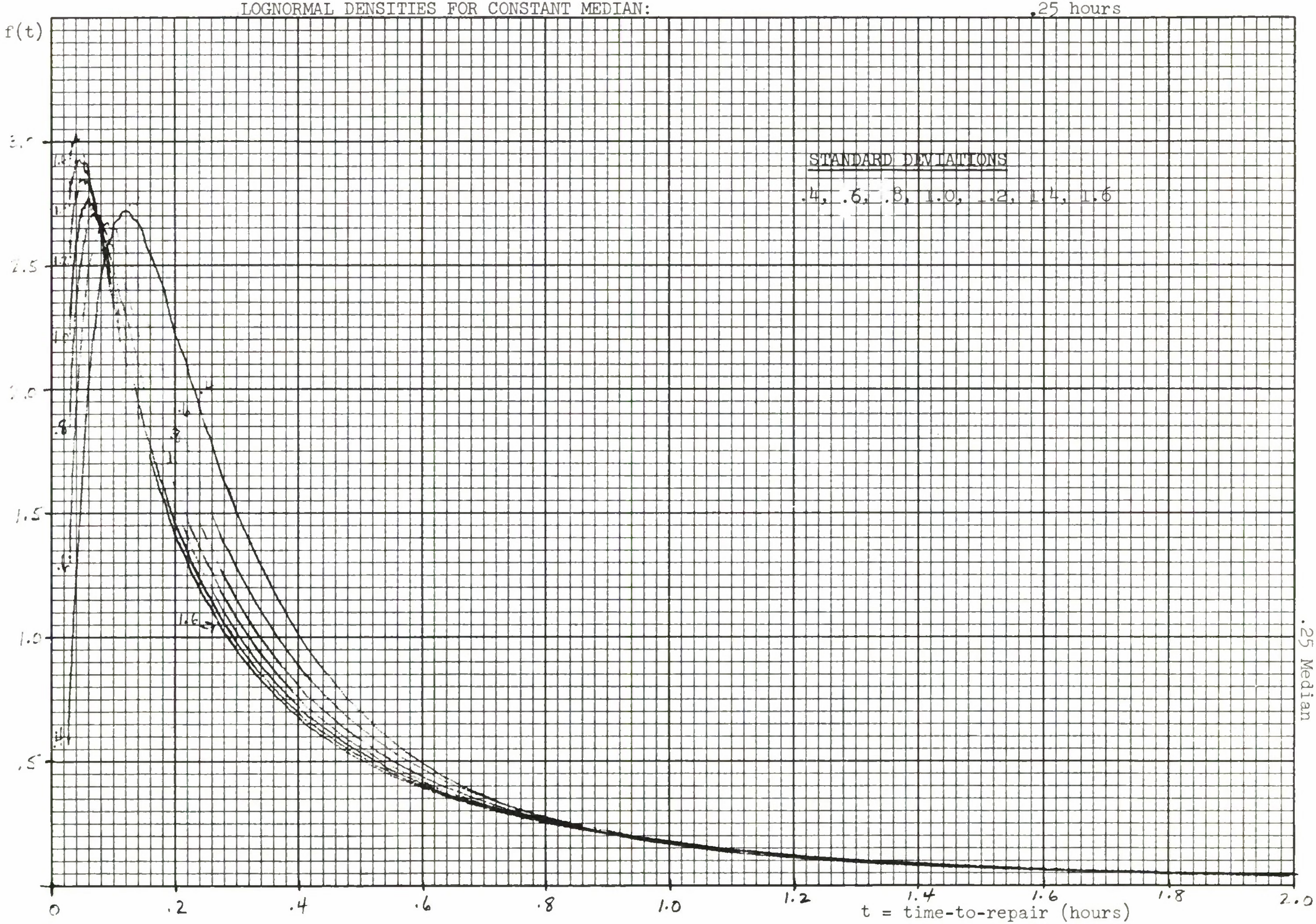
LOGNORMAL DENSITY AND CUMULATIVE DISTRIBUTION CURVES FOR CONSTANT MEDIANS

The following curves were developed by the Analog Computer Laboratory of MITRE Corporation using the mathematical definitions and equations of Chapter III (Section 2).

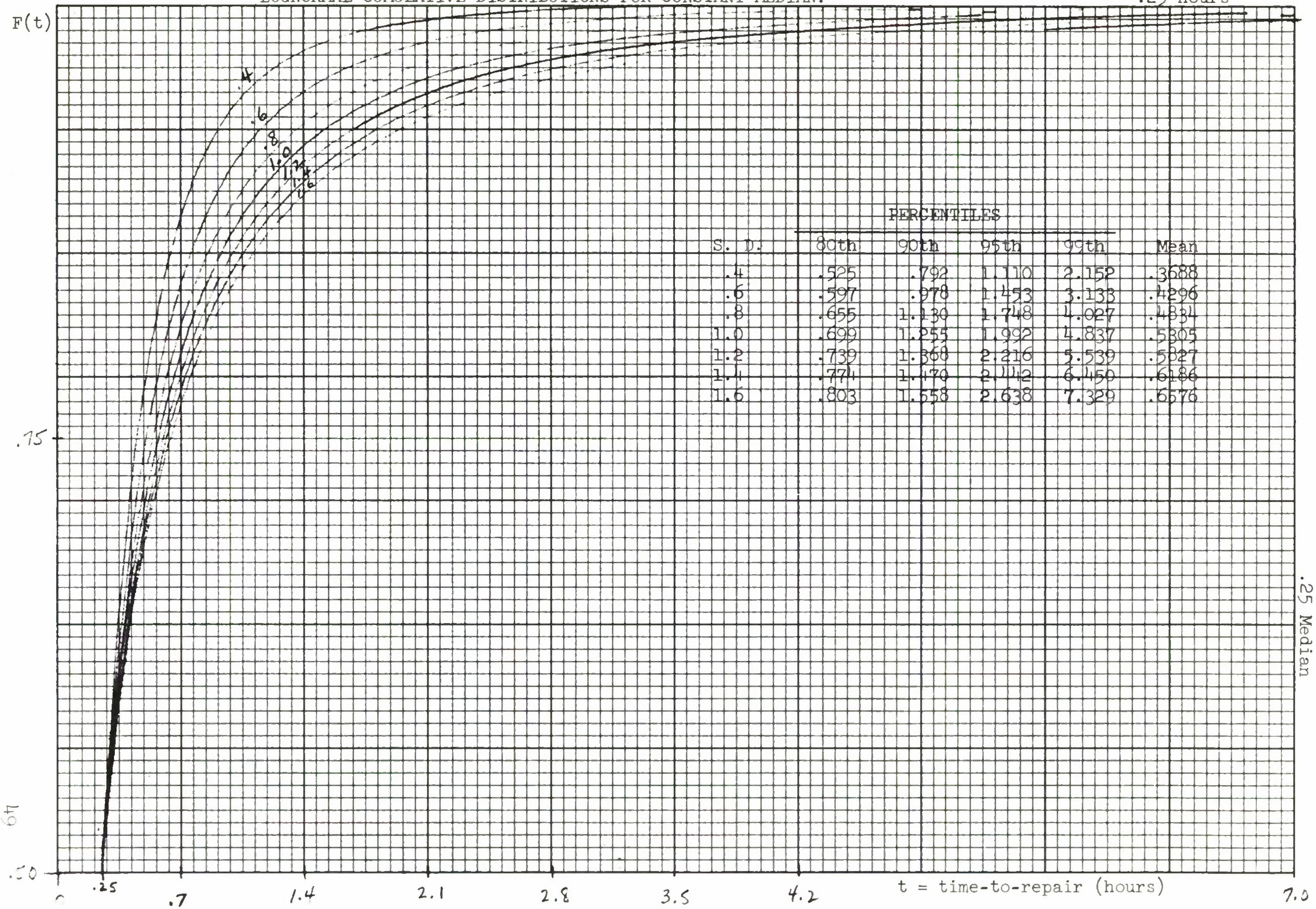
Those values which were derived by hand computations (using table look-ups) are given in the table titled "Preliminary Computations for Analog Computer", which is also included in this appendix.

The reader is cautioned to consult the horizontal and vertical scaling values whenever he enters a curve for the first time, as they usually change.

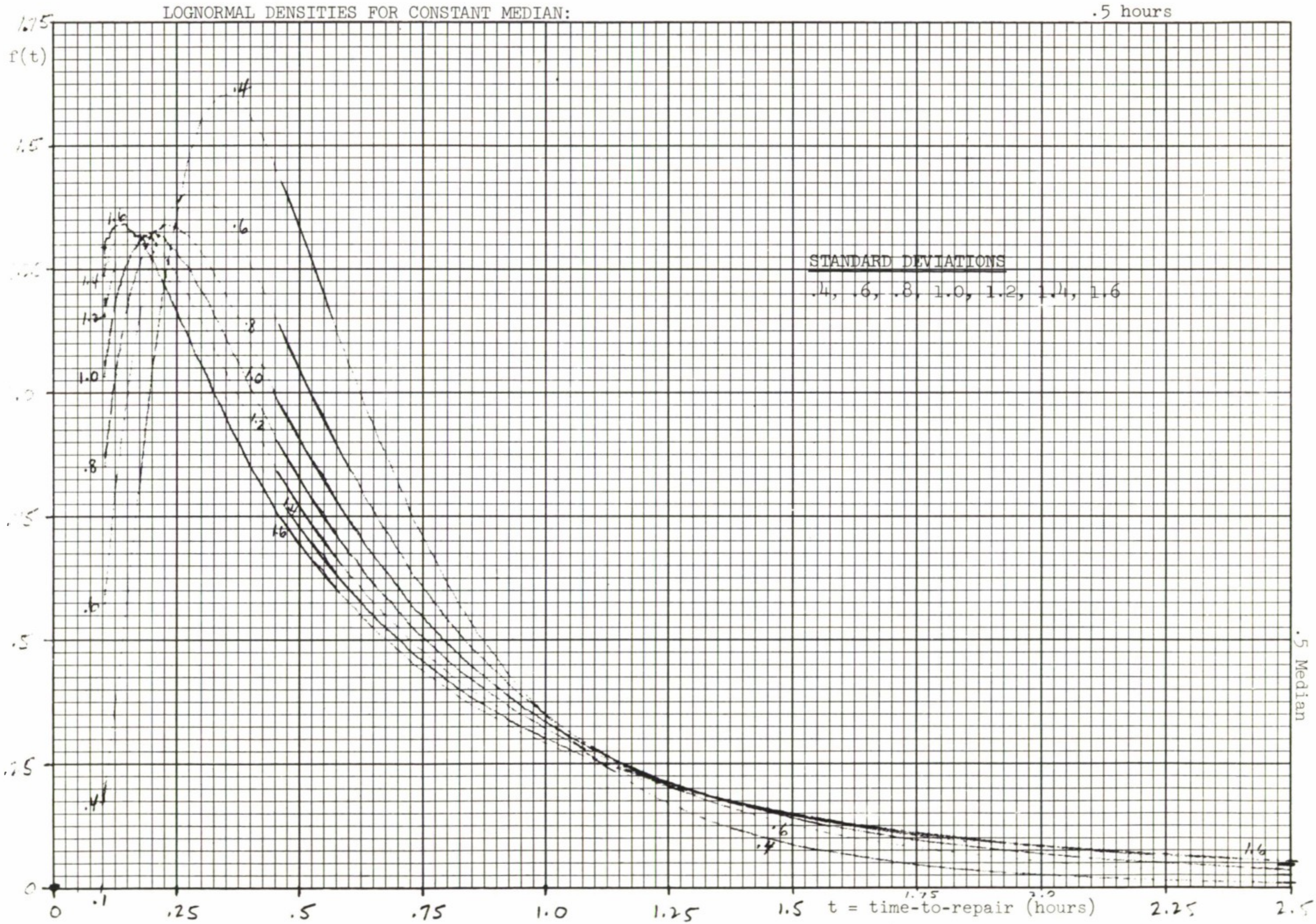
LOGNORMAL DENSITIES FOR CONSTANT MEDIAN: .25 hours



LOGNORMAL CUMULATIVE DISTRIBUTIONS FOR CONSTANT MEDIAN: .25 hours

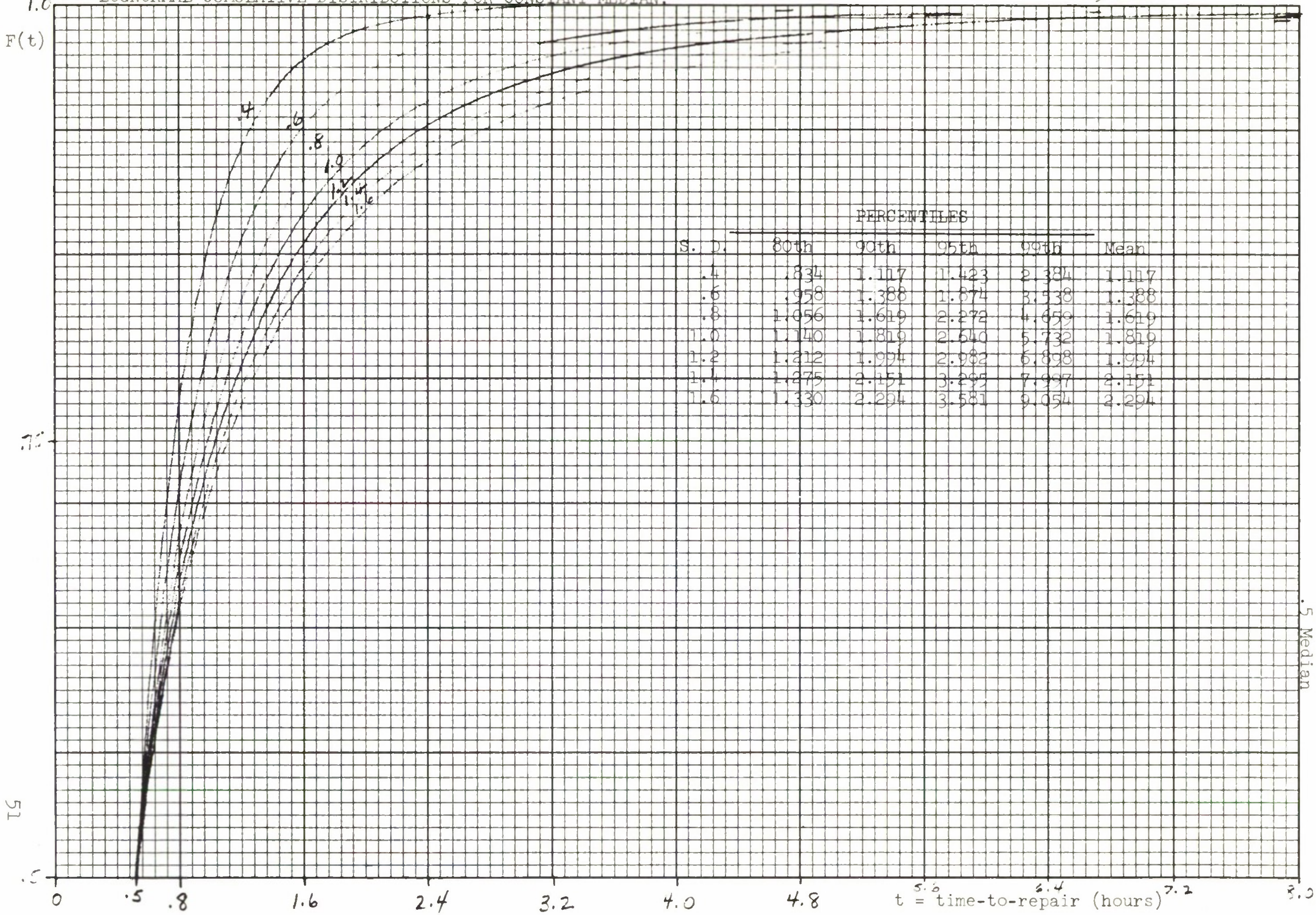


	PERCENTILES				
S. D.	80th	90th	95th	99th	Mean
.4	.525	.792	1.110	2.152	.3688
.6	.597	.978	1.453	3.133	.4296
.8	.655	1.130	1.748	4.027	.4834
1.0	.699	1.255	1.992	4.837	.5305
1.2	.739	1.368	2.216	5.539	.5827
1.4	.774	1.470	2.442	6.450	.6186
1.6	.803	1.558	2.638	7.329	.6576



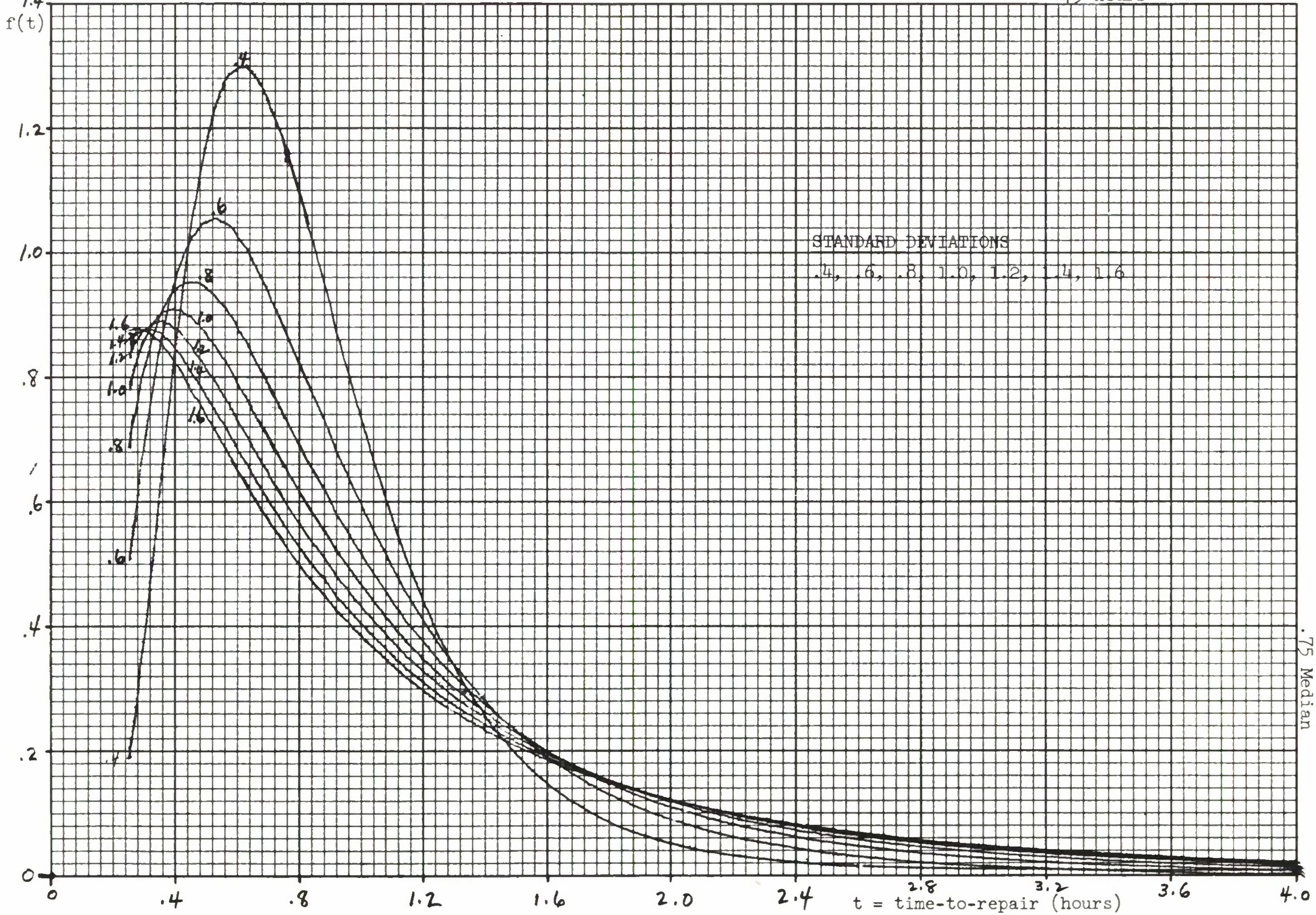
LOGNORMAL DENSITIES FOR CONSTANT MEDIAN:
.5 hours
f(t)
STANDARD DEVIATIONS
.4, .6, .8, 1.0, 1.2, 1.4, 1.6
t = time-to-repair (hours)
.5 Median
0
.25
.5
.75
1.0
1.25
1.5
2.25

LOGNORMAL CUMULATIVE DISTRIBUTIONS FOR CONSTANT MEDIAN: .5 hours

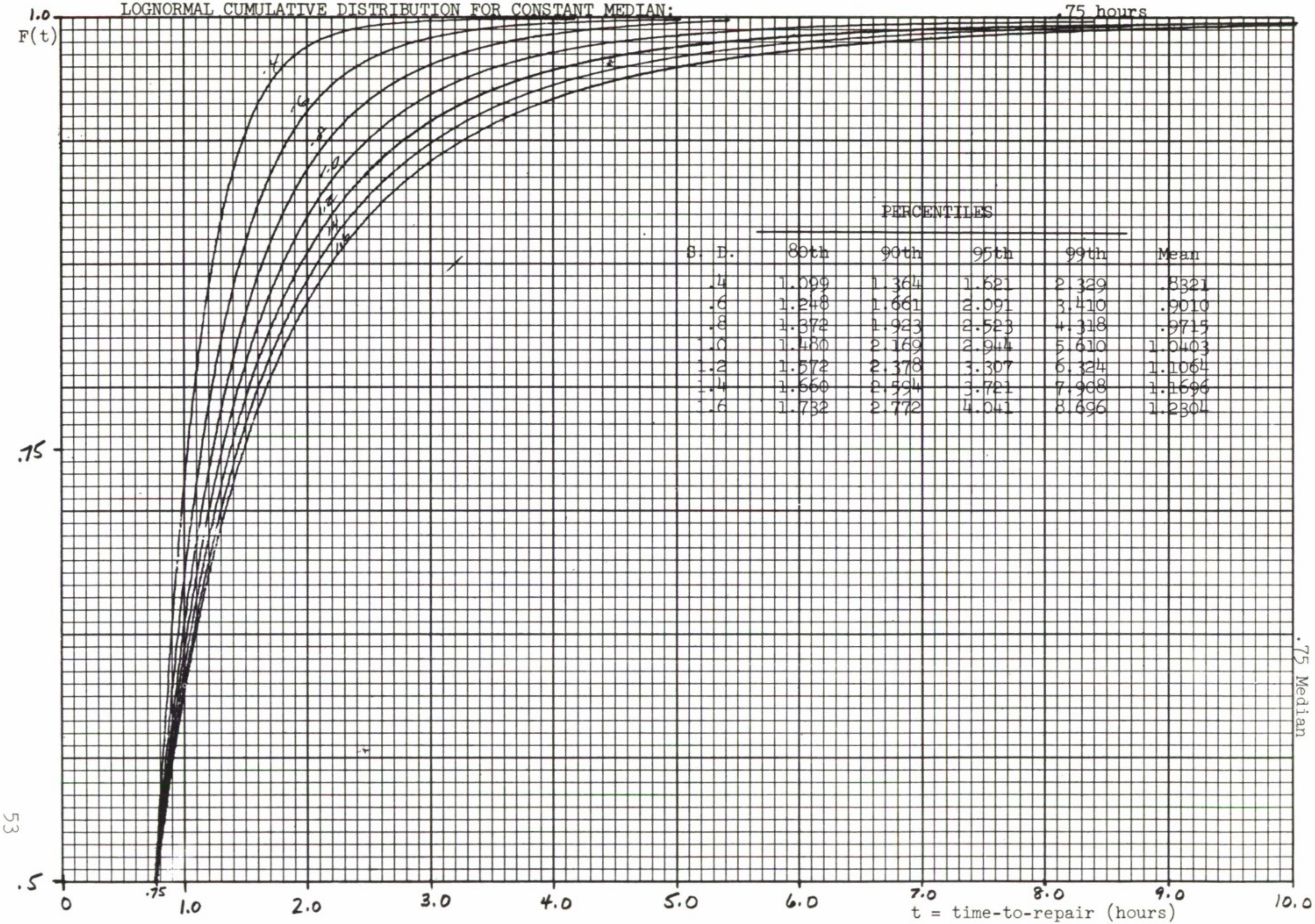


S. D.	PERCENTILES 80th	90th	95th	99th	Mean
.4	.834	1.117	1.423	2.384	1.117
.6	.958	1.388	1.874	3.538	1.388
.8	1.056	1.619	2.272	4.659	1.619
1.0	1.140	1.819	2.640	5.732	1.819
1.2	1.212	1.994	2.982	6.898	1.994
1.4	1.275	2.151	3.295	7.997	2.151
1.6	1.330	2.294	3.581	9.054	2.294

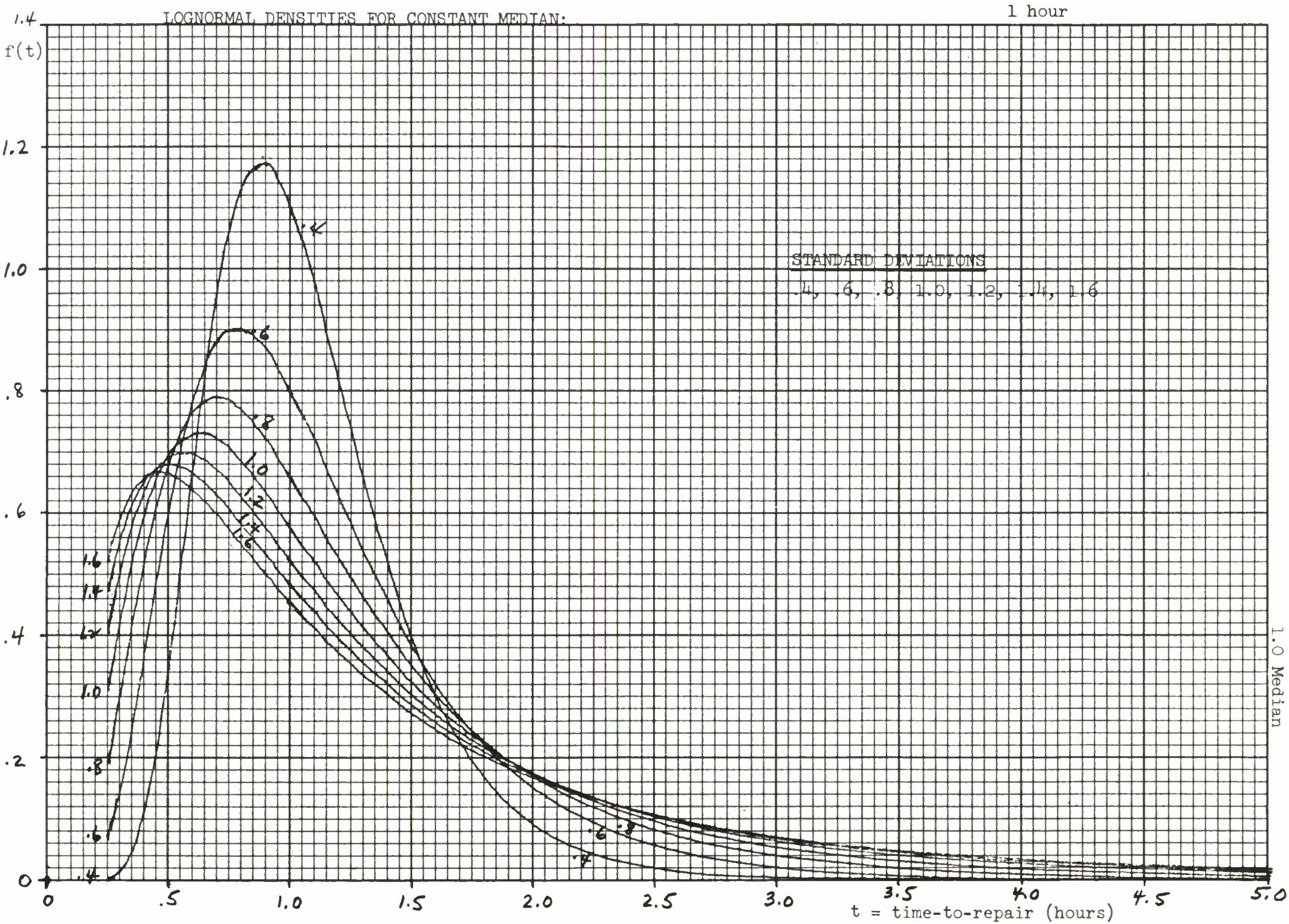
LOGNORMAL DENSITIES FOR CONSTANT MEDIAN: .75 hours



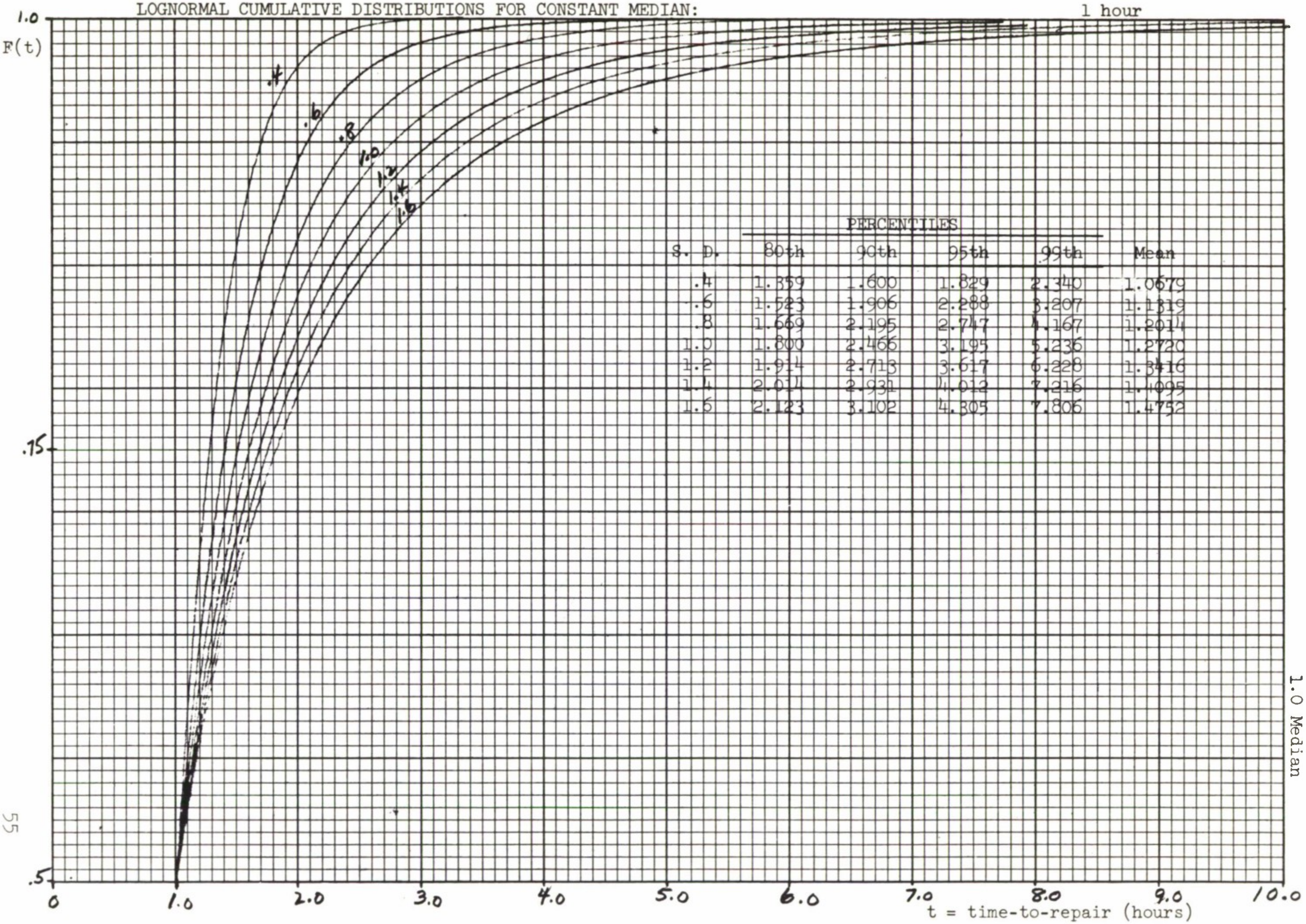
LOGNORMAL CUMULATIVE DISTRIBUTION FOR CONSTANT MEDIAN: .75 hours



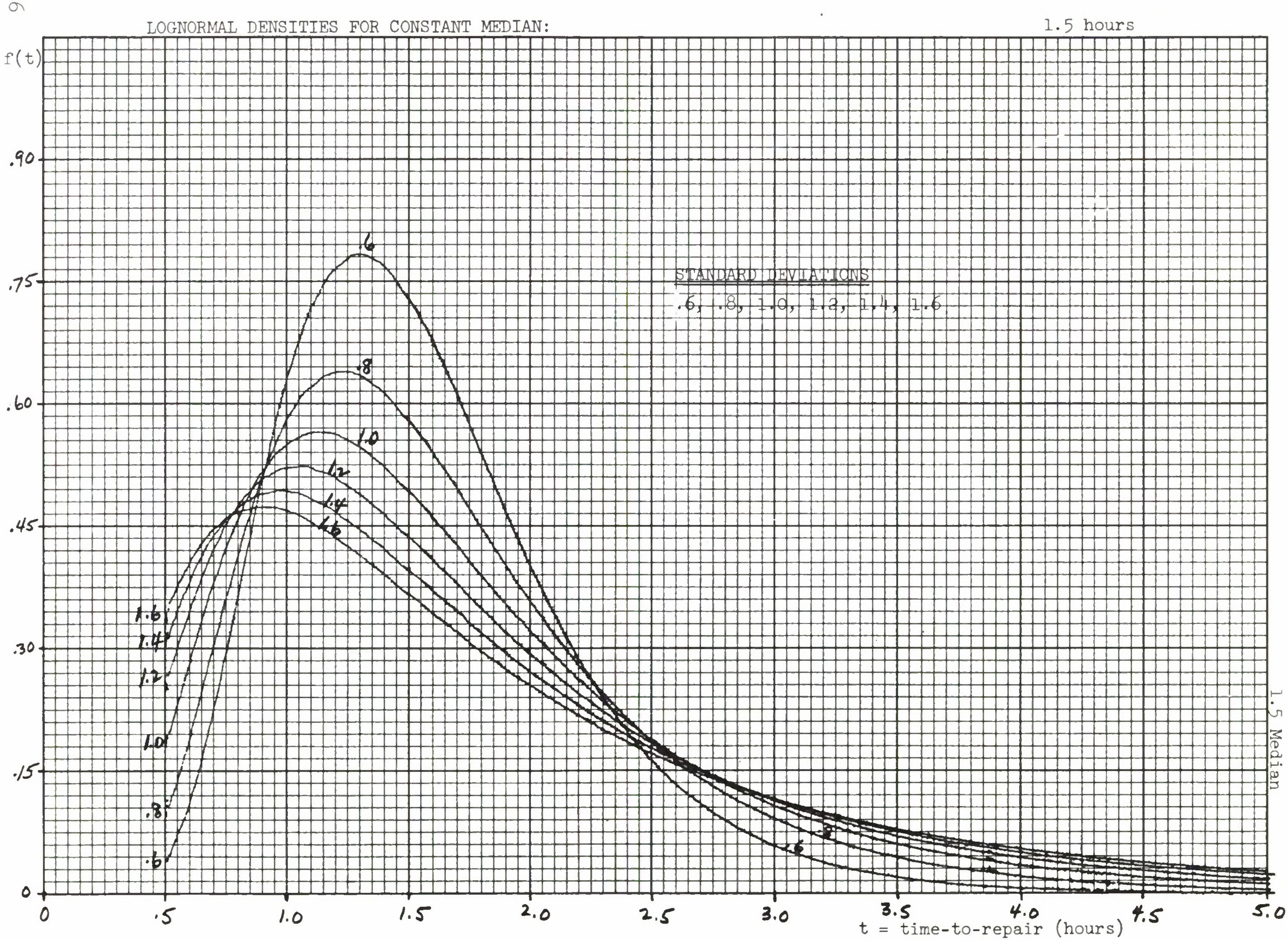
	PERCENTILES				
S. D.	80th	90th	95th	99th	Mean
.4	1.099	1.364	1.621	2.329	.8321
.6	1.248	1.661	2.091	3.410	.9010
.8	1.372	1.923	2.523	4.318	.9715
1.0	1.480	2.169	2.944	5.610	1.0403
1.2	1.572	2.378	3.307	6.324	1.1064
1.4	1.660	2.594	3.721	7.908	1.1696
1.6	1.732	2.772	4.041	8.696	1.2304


LOGNORMAL DENSITIES FOR CONSTANT MEDIAN:
1 hour
1.0 Median
STANDARD DEVIATIONS
.4, .6, .8, 1.0, 1.2, 1.4, 1.6
f(t)
t = time-to-repair (hours)

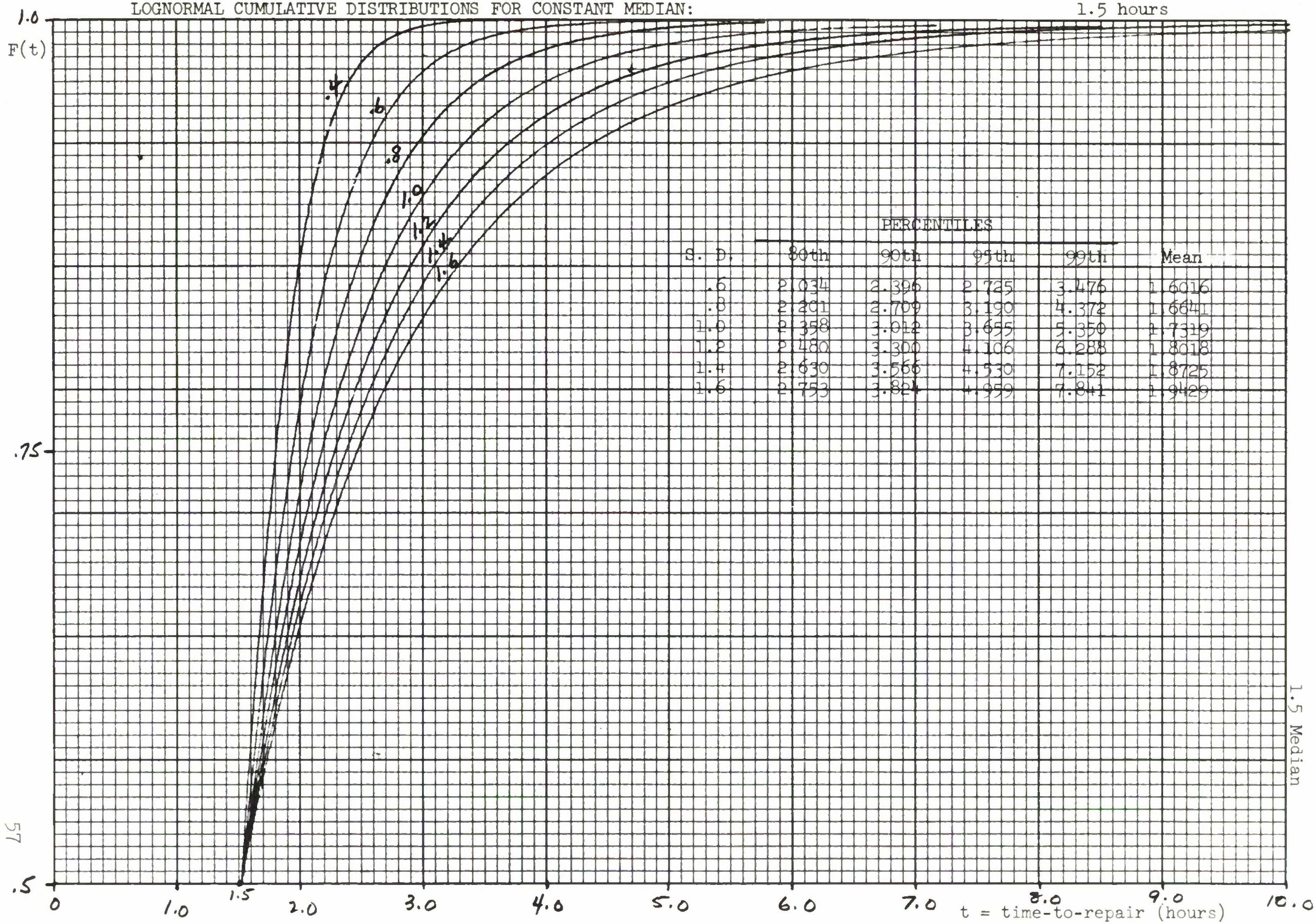
LOGNORMAL CUMULATIVE DISTRIBUTIONS FOR CONSTANT MEDIAN: 1 hour



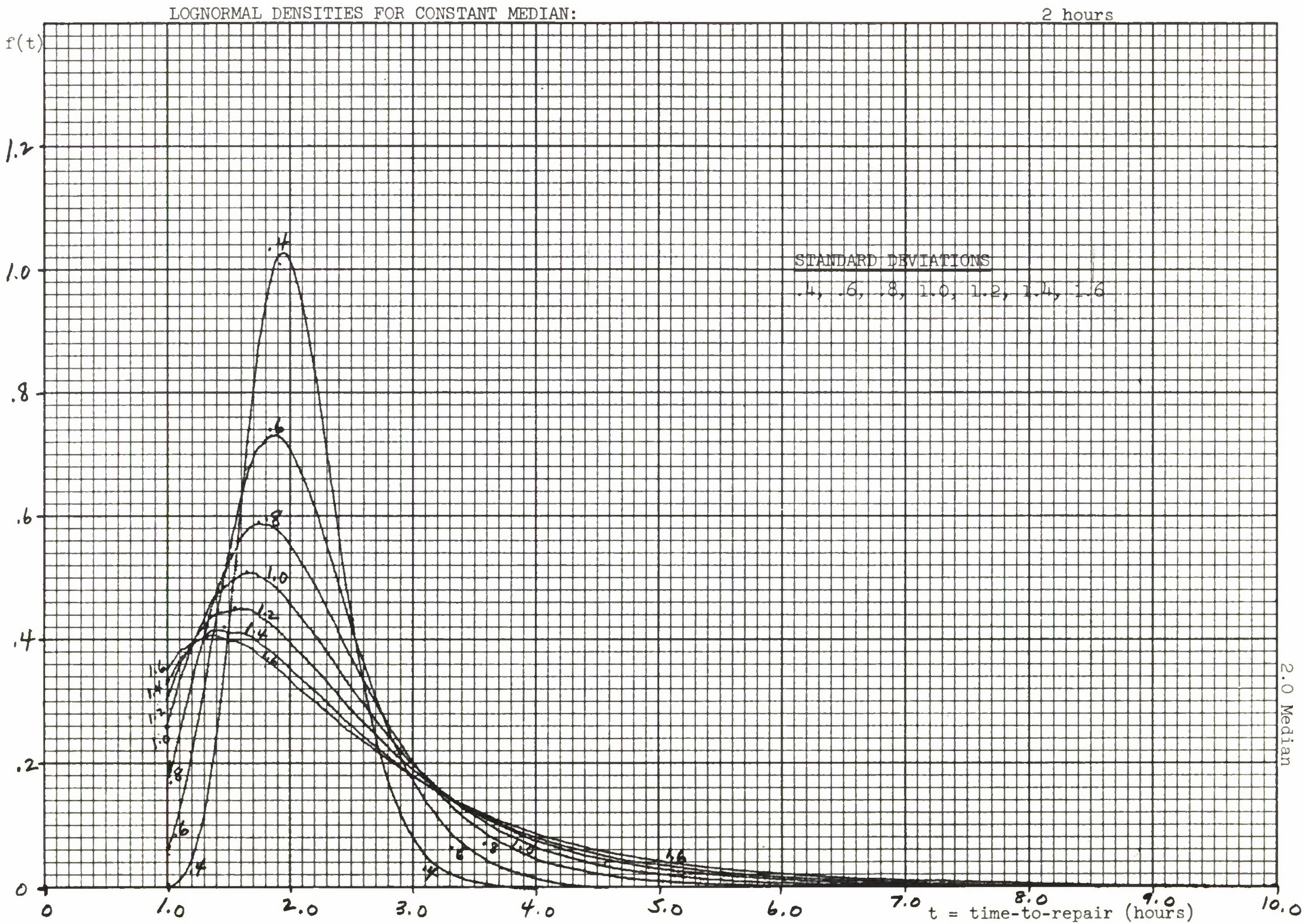
S. D.	PERCENTILES 80th	90th	95th	99th	Mean
.4	1.359	1.600	1.829	2.340	1.0679
.6	1.523	1.906	2.288	3.207	1.1319
.8	1.669	2.195	2.747	4.167	1.2014
1.0	1.800	2.466	3.195	5.236	1.2720
1.2	1.914	2.713	3.617	6.228	1.3416
1.4	2.014	2.931	4.012	7.216	1.4095
1.6	2.123	3.102	4.305	7.806	1.4752


LOGNORMAL DENSITIES FOR CONSTANT MEDIAN:
1.5 hours
f(t)
.90
.75
.60
.45
.30
.15
0
STANDARD DEVIATIONS
.6, .8, 1.0, 1.2, 1.4, 1.6
.6
.8
1.0
1.2
1.4
1.6
1.5 Median
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
t = time-to-repair (hours)

LOGNORMAL CUMULATIVE DISTRIBUTIONS FOR CONSTANT MEDIAN: 1.5 hours

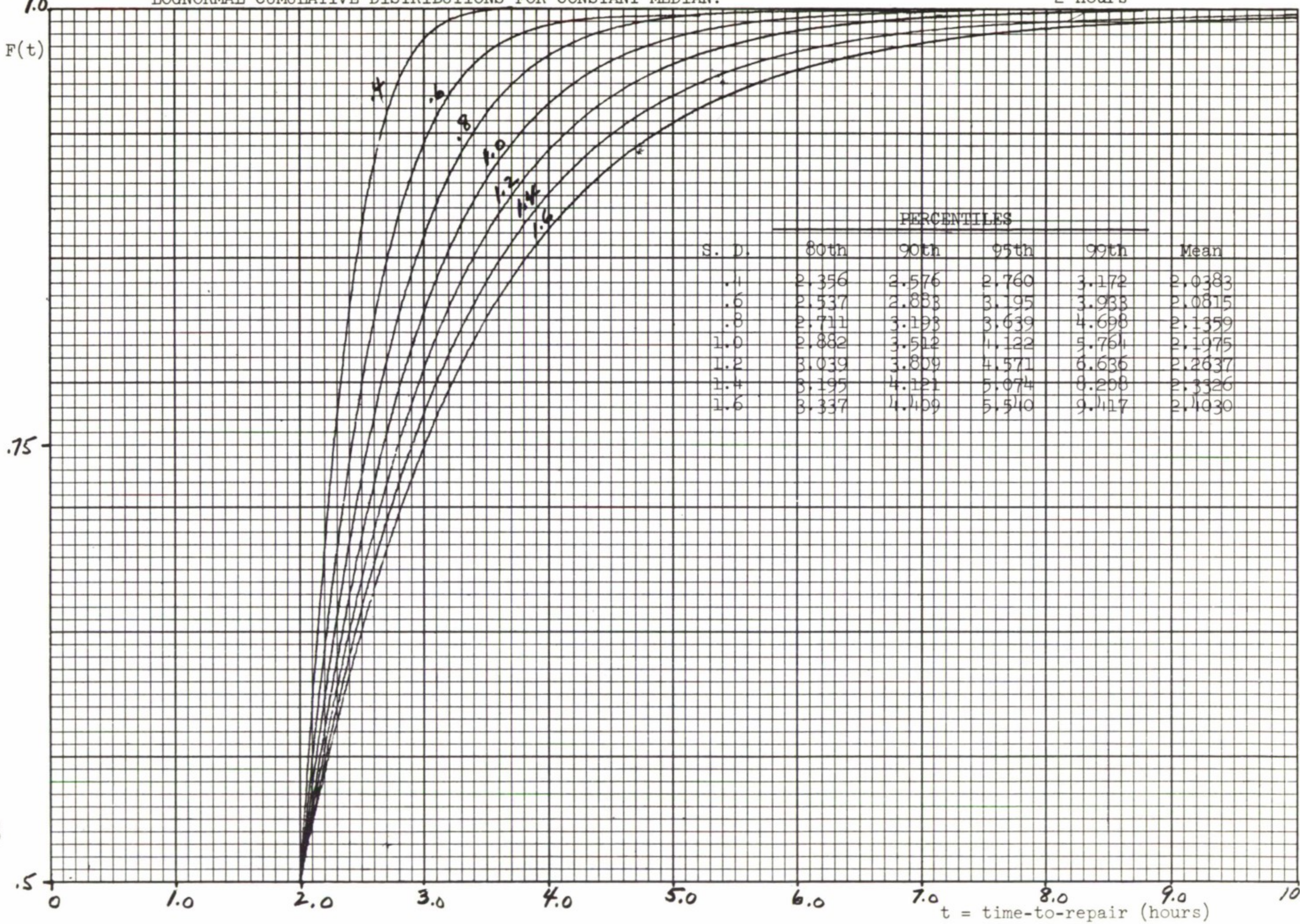


	PERCENTILES				
S. D.	80th	90th	95th	99th	Mean
.6	2.034	2.396	2.725	3.476	1.6016
.8	2.201	2.709	3.190	4.372	1.6641
1.0	2.358	3.012	3.655	5.350	1.7319
1.2	2.480	3.300	4.106	6.288	1.8018
1.4	2.630	3.566	4.530	7.152	1.8725
1.6	2.753	3.824	4.959	7.841	1.9429



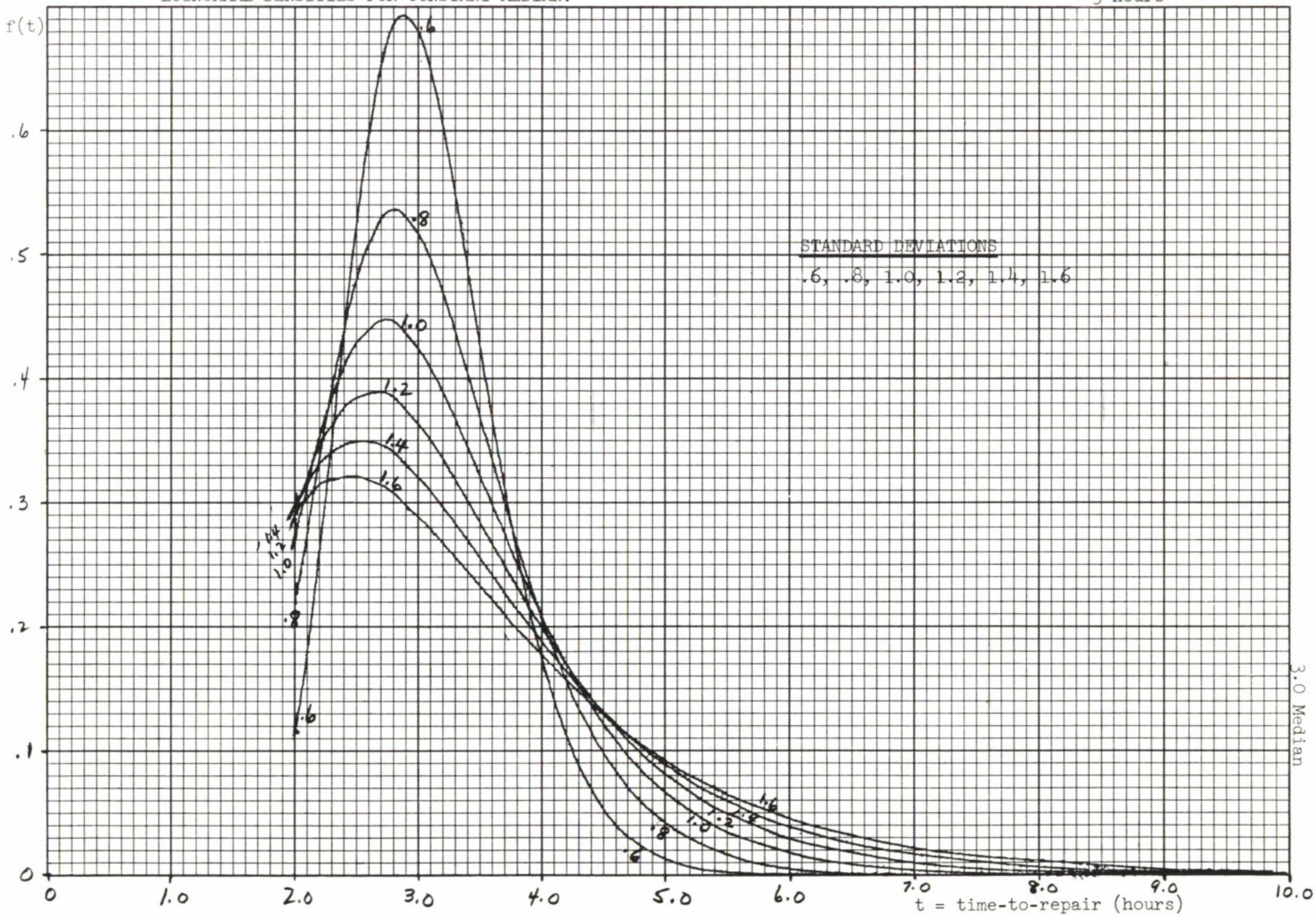
LOGNORMAL DENSITIES FOR CONSTANT MEDIAN:
2 hours
f(t)
STANDARD DEVIATIONS
.4, .6, .8, 1.0, 1.2, 1.4, 1.6
t = time-to-repair (hours)
2.0 Median

LOGNORMAL CUMULATIVE DISTRIBUTIONS FOR CONSTANT MEDIAN: 2 hours

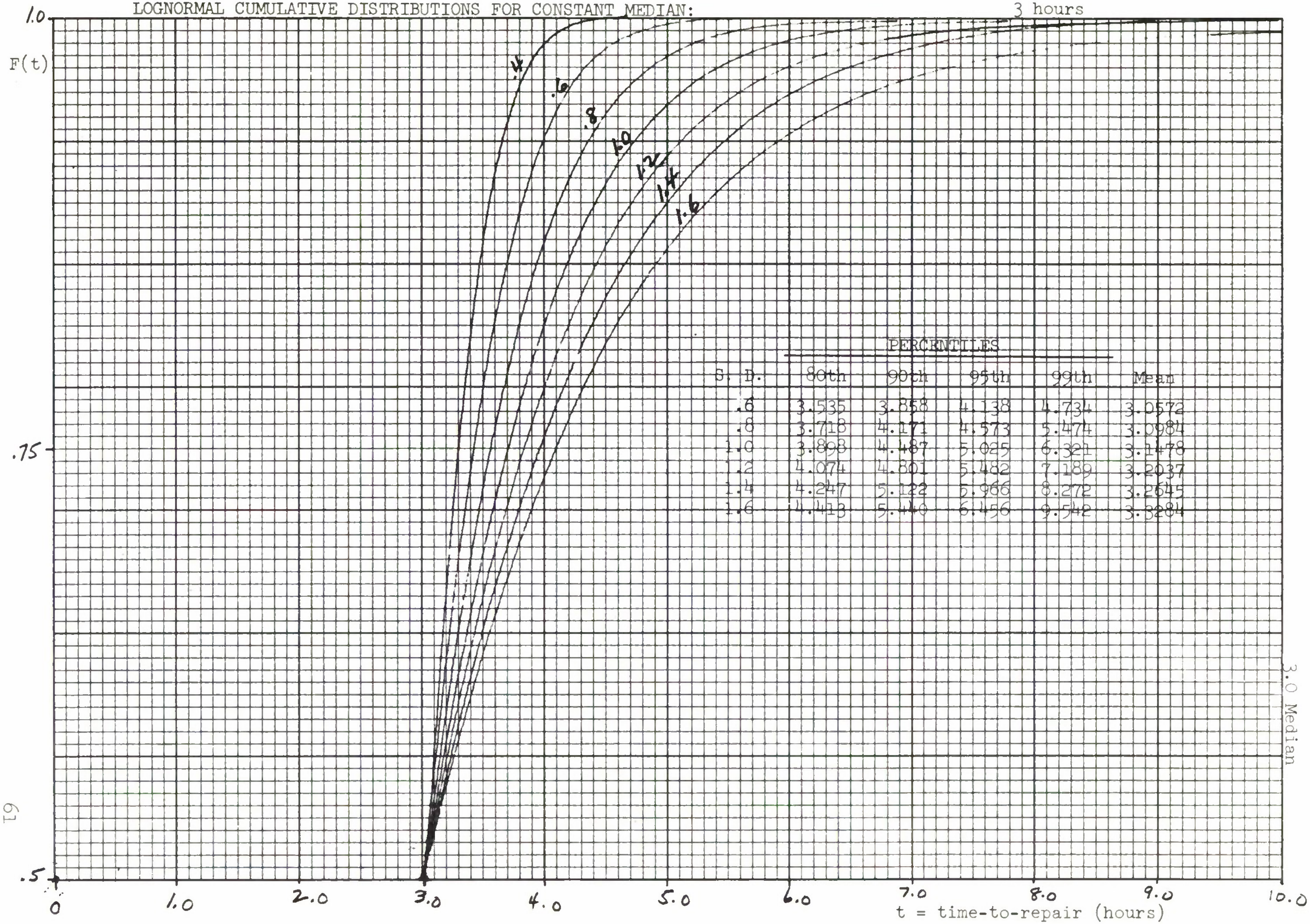


	PERCENTILES				
S. D.	80th	90th	95th	99th	Mean
.4	2.356	2.576	2.760	3.172	2.0383
.6	2.537	2.883	3.195	3.933	2.0815
.8	2.711	3.193	3.639	4.698	2.1359
1.0	2.882	3.512	4.122	5.764	2.1975
1.2	3.039	3.809	4.571	6.636	2.2637
1.4	3.195	4.121	5.074	8.208	2.3326
1.6	3.337	4.409	5.540	9.417	2.4030

LOGNORMAL DENSITIES FOR CONSTANT MEDIAN: 3 hours



LOGNORMAL CUMULATIVE DISTRIBUTIONS FOR CONSTANT MEDIAN: 3 hours



S. D.	PERCENTILES 80th	90th	95th	99th	Mean
.6	3.535	3.858	4.138	4.734	3.0572
.8	3.718	4.171	4.573	5.474	3.0984
1.0	3.898	4.487	5.025	6.321	3.1478
1.2	4.074	4.801	5.482	7.189	3.2037
1.4	4.247	5.122	5.966	8.272	3.2645
1.6	4.413	5.440	6.456	9.542	3.3284

PRELIMINARY COMPUTATIONS FOR ANALOG COMPUTER RUN

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	σ	σ^2	$\frac{1}{\theta^2}$	$\frac{\sigma^2}{\theta^2}$	$.25\frac{\sigma^2}{\theta^2}$	5	.5 + 6	ln 7	ρ = 8	$\frac{1}{\rho}$	$\frac{1}{\rho^2}$	$\frac{1}{\rho\ 2}$	$\frac{1}{2\rho^2}$	$\omega=\ln\theta$
θ = .25 hours	.05	.0025	16.00	.04	.29	.538	1.04	.04	.19	5.14	26.46	2.05	12.23	-1.386
	.10	.01		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	.20	.04		.64	.89	.94	1.44	.37	.61	1.65	2.72	.66	1.36	
	.40	.16		2.56	2.81	1.68	2.18	.78	.88	1.13	1.29	.45	.64	
	.60	.36		5.76	6.01	2.45	2.95	1.08	1.04	.96	.92	.38	.46	
	.80	.64		10.24	10.49	3.24	3.74	1.32	1.15	.87	.76	.35	.38	
	1.00	1.00		16.00	16.25	4.03	4.50	1.50	1.23	.82	.66	.33	.33	
	1.20	1.44		23.04	23.29	4.83	5.33	1.67	1.29	.77	.60	.31	.30	
	1.40	1.96		31.36	31.61	5.62	6.12	1.81	1.35	.74	.55	.30	.28	
	1.60	2.56		40.96	41.21	6.42	6.92	1.93	1.39	.72	.52	.29	.26	
θ = .50 hours	.05	.0025	4.00	.01	.26	.51	1.01	.01	.10	10.05	101.01	4.01	50.50	- .693
	.10	.01		.04	.29	.54	1.04	.04	.19	5.14	26.46	2.05	13.23	
	.20	.04		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	.40	.16		.64	.89	.94	1.44	.37	.61	1.65	2.72	.66	1.36	
	.60	.36		1.44	1.69	1.30	1.80	.59	.77	1.30	1.70	.52	.85	
	.80	.64		2.56	2.81	1.68	2.18	.78	.88	1.13	1.29	.45	.64	
	1.00	1.00		4.00	4.25	2.06	2.56	.94	.97	1.03	1.06	.41	.53	
	1.20	1.44		5.76	6.01	2.45	2.95	1.08	1.04	.96	.92	.38	.46	
	1.40	1.96		7.84	8.09	2.84	3.34	1.21	1.10	.91	.83	.36	.41	
	1.60	2.56		10.24	10.49	3.24	3.74	1.32	1.15	.87	.76	.35	.38	
θ = .75 hours	.05	.0025	1.778	.004	.25	.50	1.004	.004	.07	15.08	227.50	6.02	113.75	- .288
	.10	.01		.02	.27	.52	1.02	.02	.13	7.58	57.50	3.02	28.74	
	.20	.04		.07	.32	.57	1.07	.06	.25	3.93	15.48	1.57	7.74	
	.40	.16		.28	.53	.73	1.23	.21	.46	2.19	4.81	.88	2.41	
	.60	.36		.64	.89	.94	1.44	.37	.61	1.65	2.73	.66	1.36	
	.80	.64		1.13	1.39	1.18	1.68	.52	.72	1.39	1.93	.55	.97	
	1.00	1.00		1.78	2.03	1.42	1.92	.65	.81	1.24	1.53	.49	.76	
	1.20	1.44		2.56	2.81	1.68	2.18	.78	.88	1.13	1.29	.45	.64	
	1.40	1.96		3.48	3.73	1.93	2.43	.89	.94	1.06	1.13	.42	.56	
	1.60	2.56		4.55	4.80	2.19	2.69	.99	.995	1.01	1.01	.40	..51	
θ = 1.00 hours	.05	.0025	1.0	.0025	.25	.50	1.002	.0025	.05	20.00	400.00	7.98	200.00	0
	.10	.01		.01	.26	.51	1.01	.01	.10	10.05	101.01	4.01	50.50	
	.20	.04		.04	.29	.54	1.04	.04	.19	5.14	26.46	2.05	13.23	
	.40	.16		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	.60	.36		.36	.61	.78	1.28	.25	.50	2.01	4.04	.80	2.02	
	.80	.64		.64	.89	.94	1.44	.37	.61	1.65	2.73	.66	1.36	

	1.00	1.00		1.00	1.25	1.12	1.62	.48	.69	1.44	2.08	.58	1.04	
	1.20	1.44		1.44	1.69	1.30	1.80	.59	.77	1.30	1.70	.52	.85	
	1.40	1.96		1.96	2.21	1.49	1.99	.69	.83	1.21	1.46	.48	.73	
	1.60	2.56		2.56	2.81	1.68	2.18	.78	.88	1.13	1.29	.45	.64	
θ = 1.50 hours	.05	.0025	.444	.001	.25	.50	1.001	.001	.03	30.12	907.24	12.02	453.62	.405
	.10	.01		.004	.25	.50	1.004	.004	.07	15.24	232.38	6.08	116.19	
	.20	.04		.02	.27	.52	1.02	.02	.13	7.58	57.48	3.02	28.74	
	.40	.16		.07	.32	.57	1.07	.06	.25	3.94	15.50	1.57	7.75	
	.60	.36		.16	.41	.64	1.14	.13	.36	2.76	7.62	1.10	3.31	
	.80	.64		.28	.53	.73	1.23	.21	.46	2.19	4.81	.88	2.41	
	1.00	1.00		.44	.69	.83	1.33	2.88	.54	1.87	3.48	.74	1.74	
	1.20	1.44		.64	.89	.94	1.44	.37	.61	1.65	2.73	.66	1.36	
	1.40	1.96		.87	1.12	1.06	1.56	.44	.67	1.50	2.25	.60	1.13	
	1.60	2.56		1.14	1.39	1.18	1.68	.52	.72	1.39	1.93	.55	.97	
θ = 2.0 hours	.05	.0025	.2500	.00063	.251	.501	1.00	.001	.024	40.82	16666.0	16.28	832.98	.693
	.10	.01		.0025	.252	.503	1.00	.002	.05	20.00	400.00	7.98	200.00	
	.20	.04		.01	.26	.51	1.01	.01	.10	10.05	101.01	4.01	50.50	
	.40	.16		.04	.29	.54	1.04	.04	.19	5.14	26.46	2.05	13.23	
	.60	.36		.09	.34	.58	1.08	.08	.28	3.54	12.53	1.41	6.26	
	.80	.64		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	1.00	1.00		.25	.50	.71	1.21	.19	.43	2.31	5.31	.92	2.67	
	1.20	1.44		.36	.61	.78	1.28	.25	.50	2.01	4.04	.80	2.02	
	1.40	1.96		.49	.74	.86	1.36	.31	.55	1.80	3.25	.72	1.63	
	1.60	2.56		.64	.89	.94	1.44	.37	.61	1.65	2.72	.66	1.36	
θ = 3.0 hours	.05	.0025	.1111	.00028	.250	.500	1.00	.0003	.017	57.80	3341.2	23.06	1670.62	1.099
	.10	.01		.0011	.251	.501	1.00	.001	.033	30.12	907.24	12.02	453.62	
	.20	.04		.0044	.254	.504	1.00	.004	.066	15.08	227.50	6.02	113.75	
	.40	.16		.0178	.268	.518	1.02	.017	.132	7.60	57.83	3.03	28.92	
	.60	.36		.04	.29	.54	1.04	.038	.194	5.14	26.46	2.05	13.23	
	.80	.64		.07	.32	.57	1.07	.06	.25	3.93	15.48	1.57	7.74	
	1.00	1.00		.11	.36	.60	1.10	.10	.31	3.22	10.39	1.29	5.20	
	1.20	1.44		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	1.40	1.96		.22	.47	.68	1.18	.17	.41	2.43	5.92	.97	2.96	
	1.60	2.56		.28	.53	.71	1.23	.21	.46	2.19	4.81	.88	2.41	

AFSC FORM JUL 62 185 I   AFSC (AAFB) 1962   GENERAL PURPOSE WORK SHEET (10½" X 8")   REPLACES AFLC FORM 192G, NOV 56, WHICH IS NO LONGER APPLICABLE TO AFSC.

MEAN VALUE COMPUTATIONS

$\theta = .25$	S	ρ^2	$\frac{\rho^2}{2}$	ω	$\omega + \frac{\rho^2}{2}$	Mean $e^{\omega + \frac{\rho^2}{2}}$	
	.05	.03779	.0189	-1.3863	-1.3674	.2548	
	.1	.1313	.0657		-1.3206	.2670	
	.2	.3670	.1835		-1.2028	.3004	
	.4	.7776	.3888		- .9975	.3688	
	.6	1.0826	.5413		- .8450	.4296	
	.8	1.3188	.6594		- .7269	.4834	
	1.0	1.5048	.7524		- .6339	.5305	
	1.2	1.6726	.8463		- .5400	.5827	
	1.4	1.8120	.9060		- .4803	.6186	
	1.6	1.9343	.9672		- .4191	.6576	
$\theta = .5$							
	.05	.0099	.00495	- .6931	- .6882	.5025	
	.1	.03779	.0189		- .6742	.5096	
	.2	.1313	.0657		- .6274	.5339	
	.4	.3670	.1835		- .5096	.6007	
	.6	.5878	.2939		- .3992	.6709	
	.8	.7776	.3888		- .3043	.7376	
	1.0	.9407	.4704		- .2227	.8004	
	1.2	1.0822	.5411		- .1520	.8590	
	1.4	1.2074	.6037		- .0894	.9145	
	1.6	1.3188	.6594		- .0337	.9669	

AFSC FORM JUL 61 185A GENERAL PURPOSE WORKSHEET PREVIOUS EDITIONS OF THIS FORM ARE OBSOLETE.

MEAN VALUE COMPUTATIONS

$\theta = .75$	S	ρ^2	$\frac{\rho^2}{2}$	ω	$\omega + \frac{\rho^2}{2}$	Mean $e^{\omega + \frac{\rho^2}{2}}$	
	.05	.0044	.0022	-.2877	-.2855	.7516	
	.1	.0174	.0087		-.2790	.7565	
	.2	.06462	.03231		-.2554	.7746	
	.4	.2078	.1039		-.1838	.8321	
	.6	.3669	.1835		-.1042	.9010	
	.8	.5175	.2588		-.0289	.9715	
	1.0	.6543	.3272		+.0395	1.0403	
	1.2	.7776	.3888		+.1011	1.1064	
	1.4	.8889	.4444		+.1567	1.1696	
	1.6	.9900	.4950		+.2073	1.2304	
$\theta = 1$							
	.05	.0055	.00275	0	+.00275	1.0028	
	.1	.0099	.00495		+.00495	1.0050	
	.2	.03779	.0189		+.0189	1.0191	
	.4	.1313	.0657		+.0657	1.0679	
	.6	.2477	.1239		+.1239	1.1319	
	.8	.3669	.1835		+.1835	1.2014	
	1.0	.4812	.2406		+.2406	1.2720	
	1.2	.5878	.2939		+.2939	1.3416	
	1.4	.6864	.3432		+.3432	1.4095	
	1.6	.7776	.3888		+.3888	1.4752	

MEAN VALUE COMPUTATIONS

	S	ρ^2	$\frac{\rho^2}{2}$	ω	$\omega + \frac{\rho^2}{2}$	Mean $e^{\omega + \frac{\rho^2}{2}}$	
$\theta = 1.5$							
	.05	.0011	.00055	.4054	+.4060	1.5008	
	.1	.0043	.00215		+.4076	1.5032	
	.2	.0174	.0087		+.4141	1.5130	
	.4	.0645	.0323		+.4377	1.5491	
	.6	.1312	.0656		+.4710	1.6016	
	.8	.2078	.1039		+.5093	1.6641	
	1.0	.2875	.1438		+.5492	1.7319	
	1.2	.3668	.1834		+.5888	1.8018	
	1.4	.4437	.2219		+.6273	1.8725	
	1.6	.5175	.2588		+.6642	1.9429	
$\theta = 2$							
	.05	.0006	.0003	.6932	+.6935	2.0010	
	.1	.0025	.00125		+.6945	2.0027	
	.2	.0099	.00495		+.6982	2.0101	
	.4	.03779	.0189		+.7121	2.0383	
	.6	.07981	.03991		+.7331	2.0815	
	.8	.1313	.06565		+.7589	2.1359	
	1.0	.1882	.0941		+.7873	2.1975	
	1.2	.2476	.1238		+.8170	2.2637	
	1.4	.3076	.1538		+.8470	2.3326	
	1.6	.3670	.1835		+.8767	2.4030	

AFSC FORM JUL 61 185A GENERAL PURPOSE WORKSHEET PREVIOUS EDITIONS OF THIS FORM ARE OBSOLETE.

MEAN VALUE COMPUTATIONS

$\theta = 3$	S	ρ^2	$\frac{\rho^2}{2}$	ω	$\omega + \frac{\rho^2}{2}$	Mean $e^{\omega + \frac{\rho^2}{2}}$	
	.05	.00030	.00015	1.0986	+1.0988	3.0006	
	.1	.0011	.00055		+1.0091	3.0015	
	.2	.0044	.0022		+1.1008	3.0066	
	.4	.01729	.008645		+1.1067	3.0244	
	.6	.03779	.01890		+1.1175	3.0572	
	.8	.06462	.03231		+1.1309	3.0984	
	1.0	.09622	.04811		+1.1467	3.1478	
	1.2	.1313	.06565		+1.1643	3.2037	
	1.4	.1689	.08445		+1.1831	3.2645	
	1.6	.2078	.1039		+1.2025	3.3284	

APPENDIX B

DERIVATION OF SEQUENTIAL TEST OF PERCENTILES

The mathematical details for these tests are given excellent coverage in the following texts:

1. "Sequential Analysis" by A. Wald, John Wiley & Sons, 1947, pages 88-94.

2. "Introduction to Mathematical Statistics" by P. Hoel, John Wiley & Sons, Second Edition, pages 275-277.

Presented here are the special aspects of these tests as they apply to us, as well as formulae and computations used to develop the tests given in Chapter IV.

The general procedure is as follows:

a. Denote by p, the (unknown) percentile of any contractually specified repair-time value (such as Median-time-to-repair or Xmax) for which the desired percentile value, v, is known (v = 50% for the Median test and v = 90% for the Xmax test).

b. Choose producer's risk, α, consumer's risk, β, and two numbers p_o and p_1 such that $0 < p_1 < p_o < 1$ and v is not smaller than p_1 nor larger than p_o. We wish to decide which of the following hypotheses is correct:

   H_o: $p = p_o$

   H_1: $p = p_1$

   Wald advises us as follows: "Thus, the tolerated risks are characterized by the four numbers, p_o, p_1, α, and β. The choice of these four quantities is not a statistical problem. They will be selected on the basis of practical considerations in each particular case."

c. Once (a) and (b) are accomplished, the following formulae provide the required decision rule:

<u>Accept if</u>:

A. $$S_n \geq \frac{\ln\left(\frac{\beta}{1-\alpha}\right) + n \ln c}{\ln\left(\frac{c}{d}\right)}$$

<u>Reject if</u>:

B. $$S_n \leq \frac{\ln\left(\frac{1-\beta}{\alpha}\right) + n \ln c}{\ln\left(\frac{c}{d}\right)}$$

where "ln" denotes the natural logarithm function, and:

$n = \left\{\begin{array}{l}\text{total number of repair-time observations} \\ \text{that are made.}\end{array}\right\}$

$S_n = \left\{\begin{array}{l}\text{the number of observations that take less} \\ \text{time than the specified repair-time value} \\ \text{(corresponding to v).}\end{array}\right\}$

$$c = \frac{1 - p_o}{1 - p_1}$$

$$d = \frac{p_o}{p_1}$$ (may be called the discrimination ratio).

If neither (A) nor (B), above, hold, continue testing (take another observation) until a decision is reached.

We now apply this general procedure to derive the plan presented in Chapter IV. The following selections were made:

MEDIAN TEST v = 50th Percentile		Xmax TEST v = 90th Percentile	
p_o	= .55	p_o	= .92
p_1	= .45	p_1	= .88
α	= 10%	α	= 10%
β	= 10%	β	= 10%

Thus, the following computations were needed:

MEDIAN TEST COMPUTATIONS	Xmax TEST COMPUTATIONS
$c = \dfrac{1 - p_o}{1 - p_1} = \dfrac{9}{11}$	$c = \dfrac{1 - p_o}{1 - p_1} = \dfrac{2}{3}$
$\dfrac{c}{d} = \dfrac{c}{p_o/p_1} = \dfrac{81}{121}$	$\dfrac{c}{d} = \dfrac{c}{p_o/p_1} = \dfrac{44}{69}$
$\ln c = \ln \dfrac{9}{11} = -.2$	$\ln c = \ln\left(\dfrac{2}{3}\right) = -.41$
$\ln\left(\dfrac{c}{d}\right) = \ln\left(\dfrac{81}{121}\right) = -.4$	$\ln\left(\dfrac{c}{d}\right) = \ln\left(\dfrac{44}{69}\right) = -.45$
$\ln\left(\dfrac{\beta}{1 - \alpha}\right) = \ln\left(\dfrac{1}{9}\right) = -2.2$	$\ln\left(\dfrac{\beta}{1 - \alpha}\right) = \ln\left(\dfrac{1}{9}\right) = -2.2$
$\ln\left(\dfrac{1 - \beta}{\alpha}\right) = \ln 9 = 2.2$	$\ln\left(\dfrac{1 - \beta}{\alpha}\right) = \ln 9 = 2.2$

Using these values, the following formulae were derived:

MEDIAN TEST DECISION EQUATIONS	Xmax TEST DECISION EQUATIONS
Accept if:	Accept if:
$S_n \geqq .5n + 5.5$	$S_n \geqq .91n + 4.89$
Reject if:	Reject if:
$S_n \leqq .5n - 5.5$	$S_n \leqq .91n - 4.89$
Continue testing otherwise	Continue testing otherwise

NOTE: Normally, sequential tests are truncated by using a new decision rule which brings the decisions lines together. We did not use this technique (in Chapter IV) for the following reason: As the test continues without a decision being reached, we gain added assurance that the specified Median and Xmax values have percentile values between the selected values p_o and p_1. Although this added assurance is difficult to quantify, we feel it justifies a truncation procedure that extends the Accept region to include the Continue Test region at the end of the allotted test period. Also, we have arbitrarily decided, in Chapter IV, that decisions should not be attempted until 50 tasks have been completed.

The decision equations will actually make decisions earlier than this, if such is desired. Finally, we arbitrarily elected to end the test at $n = 100$.

There are no hard and fast rules that tell us how to make such choices. Such factors as equipment complexity, and criticality of the equipment for mission achievement must be considered, among others. We have included the decision equations so that procuring activities could design tests that suit their needs.

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13. ABSTRACT

This document develops basic concepts for treating Maintainability quantitatively, with particular attention devoted to probabilistic aspects. It focuses on the special characteristics of the Lognormal Distribution as they relate to specifying and demonstrating numerical requirements. A catalog of Lognormal curves (both density and cumulative distribution functions) are included as well as recommended accept-reject criteria for Maintainability demonstration.

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SUBJECT: Errata to ESD-TR-220; Verification of Quantitative Maintainability Requirements.

TO: Distribution

1. A copy of subject document has been sent to you by separate mailing.

2. Some columnar notation on Page 62, table entitled: "Preliminary Computation's For Analog Computer Run" were inadvertently omitted. The attached sheet is provided for direct replacement.

R.M. DeMilia

R. M. DeMilia
Staff Reliability/Maintainability
Technical Integration Division
Technical Requirements & Standards Office

1 Atch
1. Errata

PRELIMINARY COMPUTATIONS FOR ANALOG COMPUTER RUN

	(1) σ	(2) σ^2	(3) $\frac{1}{\theta^2}$	(4) $\frac{\sigma^2}{\theta^2}$	(5) $.25 \frac{\sigma^2}{\theta^2}$	(6) $\sqrt{(5)}$	(7) $.5 + (6)$	(8) $\ln (7)$	(9) $\rho = \sqrt{(8)}$	(10) $\frac{1}{\rho}$	(11) $\frac{1}{\rho^2}$	(12) $\frac{1}{\rho\sqrt{2\pi}}$	(13) $\frac{1}{2\rho^2}$	(14) $\omega = \ln\theta$
θ = .25 hours	.05	.0025	16.00	.04	.29	.538	1.04	.04	.19	5.14	26.46	2.05	12.23	-1.386
	.10	.01		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	.20	.04		.64	.89	.94	1.44	.37	.61	1.65	2.72	.66	1.36	
	.40	.16		2.56	2.81	1.68	2.18	.78	.88	1.13	1.29	.45	.64	
	.60	.36		5.76	6.01	2.45	2.95	1.08	1.04	.96	.92	.38	.46	
	.80	.64		10.24	10.49	3.24	3.74	1.32	1.15	.87	.76	.35	.38	
	1.00	1.00		16.00	16.25	4.03	4.50	1.50	1.23	.82	.66	.33	.33	
	1.20	1.44		23.04	23.29	4.83	5.33	1.67	1.29	.77	.60	.31	.30	
	1.40	1.96		31.36	31.61	5.62	6.12	1.81	1.35	.74	.55	.30	.28	
	1.60	2.56		40.96	41.21	6.42	6.92	1.93	1.39	.72	.52	.29	.26	
θ = .50 hours	.05	.0025	4.00	.01	.26	.51	1.01	.01	.10	10.05	101.01	4.01	50.50	- .693
	.10	.01		.04	.29	.54	1.04	.04	.19	5.14	26.46	2.05	13.23	
	.20	.04		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	.40	.16		.64	.89	.94	1.44	.37	.61	1.65	2.72	.66	1.36	
	.60	.36		1.44	1.69	1.30	1.80	.59	.77	1.30	1.70	.52	.85	
	.80	.64		2.56	2.81	1.68	2.18	.78	.88	1.13	1.29	.45	.64	
	1.00	1.00		4.00	4.25	2.06	2.56	.94	.97	1.03	1.06	.41	.53	
	1.20	1.44		5.76	6.01	2.45	2.95	1.08	1.04	.96	.92	.38	.46	
	1.40	1.96		7.84	8.09	2.84	3.34	1.21	1.10	.91	.83	.36	.41	
	1.60	2.56		10.24	10.49	3.24	3.74	1.32	1.15	.87	.76	.35	.38	
θ = .75 hours	.05	.0025	1.778	.004	.25	.50	1.004	.004	.07	15.08	227.50	6.02	113.75	- .288
	.10	.01		.02	.27	.52	1.02	.02	.13	7.58	57.50	3.02	28.74	
	.20	.04		.07	.32	.57	1.07	.06	.25	3.93	15.48	1.57	7.74	
	.40	.16		.28	.53	.73	1.23	.21	.46	2.19	4.81	.88	2.41	
	.60	.36		.64	.89	.94	1.44	.37	.61	1.65	2.73	.66	1.36	
	.80	.64		1.13	1.39	1.18	1.68	.52	.72	1.39	1.93	.55	.97	
	1.00	1.00		1.78	2.03	1.42	1.92	.65	.81	1.24	1.53	.49	.76	
	1.20	1.44		2.56	2.81	1.68	2.18	.78	.88	1.13	1.29	.45	.64	
	1.40	1.96		3.48	3.73	1.93	2.43	.89	.94	1.06	1.13	.42	.56	
	1.60	2.56		4.55	4.80	2.19	2.69	.99	.995	1.01	1.01	.40	..51	
θ = 1.00 hours	.05	.0025	1.0	.0025	.25	.50	1.002	.0025	.05	20.00	400.00	7.98	200.00	0
	.10	.01		.01	.26	.51	1.01	.01	.10	10.05	101.01	4.01	50.50	
	.20	.04		.04	.29	.54	1.04	.04	.19	5.14	26.46	2.05	13.23	
	.40	.16		.16	.41	.64	1.14	.13	.36	2.76	7.61	1.10	3.81	
	.60	.36		.36	.61	.78	1.28	.25	.50	2.01	4.04	.80	2.02	
	.80	.64		.64	.89	.94	1.44	.37	.61	1.65	2.73	.66	1.36	

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